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Bayes Factors for Forensic Decision Analyses with R

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Published with the support of the Swiss National Science Foundation (Grant no. 10BP12_208532/1)



ISSN 1431-875X

ISSN 2197-4136 (electronic)

Springer Texts in Statistics

ISBN 978-3-031-09838-3

ISBN 978-3-031-09839-0 (eBook)

<https://doi.org/10.1007/978-3-031-09839-0>

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To our families

Preface

The introduction of scientific evidence in legal proceedings raises a host of intricate questions and themes, ranging from the architecture of legal systems across contemporary jurisdictions and psychological aspects of judgment and decision-making, to principles and methods of logical reasoning and decision-making under uncertainty. Over decades of theoretical and practice-oriented research, scholars in fields such as law, statistics, history, philosophy of science, psychology, and forensic science have come to the understanding that the sound use of scientific findings in evidence and proof processes critically depends on the ability of forensic scientists to use formal methods of reasoning, so as to ensure a coherent approach to dealing with and communicating about uncertainty. The focal point of these developments is the recognition of probability as the reference method for measuring uncertainty.

It is thus hardly surprising that, in recent years, the intersection between law and forensic science has seen an increase in the number of reports, guidelines, and recommendations issued by eminent societies, review panels, and expert groups that insist on the importance of aligning the interpretation of scientific evidence by forensic scientists to a probabilistic measure of the value of evidence.¹ This measure is the likelihood ratio and has been widely described in peer-reviewed articles and textbooks.

What is less often recognized, however, is that the likelihood ratio is merely a particular instance of a more general concept, known as the *Bayes factor*. While the likelihood ratio is typically presented in the focused context of evidence-based discrimination between pairs of competing propositions, the Bayes factor is a method of choice for approaching a more comprehensive collection of problems commonly associated with the use of measurements and data in forensic science.

¹ Examples include documents issued by the Royal Statistical Society (Aitken et al., 2010), The Royal Society of Edinburgh (Nic Daéid et al., 2020), The UK Forensic Science Regulator (Tully, 2021), The European Network of Forensic Science Institutes (Willis et al., 2015), The Association of Forensic Science Providers (Association of Forensic Science Providers, 2009), and expert communities, in particular sub-fields of forensic science, such as forensic genetics (e.g. Gill et al., 2018) or forensic voice comparison (Drygajlo et al., 2015; Morrison et al., 2021).

Examples include the comparison of probabilistic models, model selection, and decision-making regarding competing theories and model parameters. We believe that by becoming acquainted with Bayes factors across a range of different applications, forensic scientists can strengthen the use of probabilistic methods in their respective disciplines. Forensic scientists should also gain an understanding of the role of Bayes factors in coherent decision-making under uncertainty. The core idea of this book on Bayes factors, the first on this theme in forensic science, is to address these questions.

Bayes Factors for Forensic Decision Analyses with R is a new Bayesian modeling book that provides a self-contained account of essential elements of computational Bayesian statistics using R, a leading programming language and a freely available software environment for statistical computing. This book features a well-rounded approach to three naturally interrelated topics. The first is probabilistic inference. As a core concept of Bayesian inferential statistics, Bayes factors are ideally suited to help forensic scientists think about the logical and balanced evaluation of the value of evidence. This is a necessary preliminary to coherent reporting on scientific evidence. Second, this book highlights the logical connection between probabilistic reasoning, using Bayes factors, and decision analysis under uncertainty. This perspective involves the decision-theoretic (re-)conceptualization of questions that, in classical statistics, are often framed as problems of hypothesis testing using a disparate set of concepts, such as p-values, that have a longstanding and well-documented history of misinterpretations by both scientists and recipients of expert information. Here, Bayes factors provide a sound and defensible alternative. The third theme that this book covers is operational relevance. Thus, throughout this book, all key concepts are systematically illustrated with hands-on examples and complete template code in R, including sensitivity analyses and explanations on how to interpret results in context. This usefully complements the theoretical and philosophical justifications for the coherent approach to inference and decision emphasized throughout this book.

Besides explaining the role of the Bayes factor as a guide to reasoning and as a preliminary to coherent decision analysis, the original contribution of this book is to work out the relevance of these topics with respect to two main forensic areas of application: investigation and evaluation. The first, investigation, refers to discriminating between general propositions of interest, i.e., when no named person (or object) is available for comparative examinations with a given trace, mark, or impression of unknown source. The second, evaluation, is concerned with assessing the meaning of evidence with respect to specific propositions of interest, e.g., whether given trace material, a mark, or an impression comes from a particular person (or object), rather than from an unknown person (or object). While investigation and evaluation pertain to distinct procedural phases with specific needs and constraints, they involve inferential and decisional tasks that have common conceptual underpinnings that can be formally captured, analyzed, and expressed in terms of Bayes factors, and embedded in a coherent framework for decision analysis.

This book does not contain recipes nor does it intend to prescribe what scientists should do. Instead, the aim of this book is to provide forensic scientists with

a sound analytical framework for inference and decision analysis that allows them to critically rethink their current approaches drawn from more traditional courses in probability and statistics. As prerequisites, readers should have a minimal background in probability and statistics including, ideally, notions from Bayesian statistics. With its balanced presentation of theoretical and philosophical background, together with practical illustrations, this concise book seeks to make an original contribution to forensic science literature. It will be of equal interest to forensic practitioners and applied forensic statisticians, and can be used to support courses on Bayesian statistics for forensic scientists. Occasionally, we will refer to datasets and computational routines, available as online supplementary materials on the book's website at <http://link.springer.com/>.

This book presents materials developed through a longstanding collaboration between the authors. Their research was supported, at various instances, by the *Swiss National Science Foundation*, the *Foundation for the University of Lausanne* (Fondation pour l'Université de Lausanne), the *Vaud Academic Society* (Société Académique Vaudoise), the *Department of Economics of Ca' Foscari University of Venice*, and the *School of Criminal Justice of the University of Lausanne*. The authors are deeply indebted to Colin Aitken and Daniel Ramos for their valuable advice, to Lorenzo Gaborini for sharing routines developed in his Ph.D thesis, and to Luc Besson, Jacques Linden, Raymond Marquis, Valentin Scherz, and Matthieu Schmittbuhl for sharing data of forensic interest. Finally, students and fellow researchers at *Ca' Foscari University of Venice* and the *University of Lausanne* have provided the authors with exciting and encouraging environments without which much of the writing of this book would not have been possible.

Venice, Italy
Lausanne-Dorigny, Switzerland
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August 2022

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