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Ramji Lal

Algebra 4

Lie Algebras, Chevalley Groups, and their Representations

Springer

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Ramji Lal
University of Allahabad
Prayagraj, Uttar Pradesh, India

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Dedicated to the memory of
my younger sister
(Late) Smt Pushpa (Malti),
who left us at the age of 30.

Preface

The present volume, Algebra 4, in this series of books on Algebra, centers around the study of Lie algebras, Chevalley groups, and their representation theory. Lie groups and Lie algebras are very intrinsically related. The origin of Lie groups and Lie algebras lies in the study of geometric spaces with the very crucial observation that a geometric space is determined by the group of its continuous symmetries. Lie groups and Lie algebras play a very fundamental role in Physics also.

The main concerns in the book are the following:

1. The structure theory and the classification of semi-simple Lie algebras over $\mathbb{C}$ through root space decomposition, root systems, and Dynkin diagrams.
2. The representation theory of semi-simple Lie algebras including the theorem of Harish-Chandra and the theorems of Ado and Iwasava.
3. Chevalley groups including the twisted finite simple groups of Lie types.
4. The representation theory of Chevalley groups including the Steinberg characters, Principal and Discrete series representations, and an introduction to the Deligne–Lusztig characters.

The book can act as a text for graduate and advanced graduate students specializing in the field.

There is no prerequisite essential for the book except for some basics in algebra (as in Algebra 1 and Algebra 2) together with some amount of calculus and topology. An attempt to follow the logical ordering has been made throughout the book.

My teacher: (Late) Prof. B. L. Sharma; my colleagues at the University of Allahabad; my friends: Prof. Satyadeo, Prof. S. S. Khare, Prof. H. K. Mukherji, and Dr. H. S. Tripathi; my students: Prof. R. P. Shukla, Prof. Shivdatt, Dr. Brajesh Kumar Sharma, Mr. Swapnil Srivastava, Dr. Akhilesh Yadav, Dr. Vivek Jain, Dr. Vipul Kakkar, and Dr. Laxmikant; and above all the mathematics students of Allahabad University had always been a motivating force for me to write a series of books on various topics in Algebra. Without their continuous insistence, it would have not come in the present form. I wish to express my warmest thanks to all of them.

Harish-Chandra Research Institute Allahabad has always been a great source for me to learn more and more mathematics. I wish to express my deep sense of appreciation and thanks to HRI for providing me with all the infrastructural facilities to write these volumes.

Last but not least, I wish to express my thanks to my wife Veena Srivastava who had always been helpful in this endeavor.

In spite of all the care, some mistakes and misprints might have crept and escaped my attention. I shall be grateful to any such attention. Criticisms and suggestions for the improvement of the book will be appreciated and gratefully acknowledged.

Prayagraj, India Ramji Lal
September 2020

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