

Jaya P. N. Bishwal

# Parameter Estimation in Stochastic Volatility Models

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Jaya P. N. Bishwal   
Department of Mathematics and Statistics  
University of North Carolina at Charlotte  
Charlotte, NC, USA

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To the memory of my late grand parents  
and to my parents and brothers for their love and affection

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