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Morphological Models of Random Structures

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Dedicated to my wife Anne-Marie,
to my children François, Guillaume,
Clotilde and Bénédicte
In memory of my parents

Preface

This book covers methods of Mathematical Morphology to model and simulate random sets and functions (scalar and multivariate).

These models concern many physical situations in heterogeneous media, where a probabilistic approach is required: fracture statistics of materials, scaling up of permeability in porous media, electron microscopy images (including multispectral images), rough surfaces, multicomponent composites, biological tissues, and also textures for image coding and synthesis... The common feature of these random structures is their domain of definition in n dimensions (with $n \geq 3$), requiring more general models than standard Stochastic Processes.

The present book is based on my own research developments and applications, most of them being available as papers in journals and proceedings. It develops various models of random structures available for applications, and details their probabilistic properties. A unified approach is followed in random structure modeling and simulations. The book covers all steps of modeling. Some topics detailed here are missing in previous books limited to random sets: models of scalar and multivariate random functions, multiscale models, use of random models to predict the physical behavior of microstructure (like effective properties, or fracture statistics). Concerning applications given to illustrate the theory, they are based on quantitative image analysis made on representative samples.

The main topics of the present book cover an introduction to the theory of random sets, random space tesselations, Boolean random sets and functions, space-time random sets and functions (Dead Leaves, Alternate Sequential models, Reaction-Diffusion), prediction of effective properties of random media, and probabilistic fracture theories. The book details the construction of models, their main probabilistic properties, and their practical use from experimental data by means of examples of application.

This book collects results of near 50 years of research in the area of random media, which are widely dispersed in scientific publications in various areas, and in lecture notes. In addition, some unpublished new results are provided. It is intended to make available to the scientific community tools of research in the area probabilistic modeling. It will be of interest for researchers and research engineers in the areas of applied mathematics, image analysis, and applications of models of random structures. It will be greatly profitable to theoreticians and practitioners of the simulation and prediction of physical properties of heterogeneous microstructures, as encountered in heterogeneous natural or man-made materials or in life sciences. It will be a source of inspiration for further research in these fields. Graduate students and teachers in applied probability will learn developments on the theory and applications of random structures.

Fontainebleau, *Dominique Jeulin*
August 30th 2020

Acknowledgements

A large part of the work presented in this book is the result of interactions with many persons. There is indeed a crucial impact of scientific meetings and of cooperation with colleagues which I want to acknowledge.

My first steps in applied probability, Mathematical Morphology and Geostatistics started at the beginning of 70s, thanks to the teaching of P. Formery, G. Matheron and J. Serra in Paris School of Mines. They transmitted to me the basic knowledge and the foundations for my work in research. Furthermore, they communicated their enthusiasm at the very beginning of these new disciplines. Later on, I had the chance to cooperate closely for years with G. Matheron and J. Serra in Centre de Morphologie Mathématique.

My first position was in Moscow State University by E.N. Kolomenski, to initiate a morphological analysis of microstructures and multivariate statistical analysis of geological data. Student in Mathematical Statistics with J. P. Benzécri, I learned multivariate data analysis. Then, I made a ten years research for steel industry in IRSID, starting with a work on iron ore mineralogical textures and their physical behavior in the blast furnace. The topic was wide and complex, but I enjoyed absolute trust from my management, mainly M. Schneider and M. Olette, to carry out a large European project.

I was introduced to the field of Physics of random media by E. Kröner and J. Willis, when they visited me in IRSID in 1980. I was invited to CMDS (Continuum Models and Discrete Systems) meetings and to many other Conferences, where I met and got acquainted with scientists to whom I am indebted for their strong influence, like M. Beran, D. Bergman, M.

Kachanov, S. Kanaun, K. Markov, G. Milton, M. Ostoja-Starzewski, P. Ponte Castaneda, P. Suquet, among others. A. Zaoui and T. Bretheau introduced me to the French community in Micromechanics. A probabilistic lighting was developed over years in this area through the Mecamat association and other groups, in which I cooperated with many colleagues, like M. Bornert, A. Fanget, D. François, F. Hild, F. Montheillet, S. Roux, H. Trumel, and in the Alea working group with K. Sab C. Soize.

In the area of Image Analysis and Sterology, I was an active member of ISS (International Society for Stereology), where I worked with J.L. Chermant and M. Coster, since 1976. I had many exchanges with J. Bertram, B.V. Vedel Jensen, M. Jourlin and R.E. Miles. I would like to thank my German colleagues, for a long-standing cooperation: D. Stoyan (Freiberg), K. Schladitz and C. Redenbach (Kaiserslautern), J. Ohser (Darmstadt).

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I was welcomed in Paris School of Mines in 1986 by F. Mudry, with the mission of building bridges between Mathematical Morphology and Materials Science, which is the main topic of this book. This was made possible by a close cooperation and interaction, mainly with my colleagues of three research centers in Paris School of Mines, which I want to warmly thank: J. Angulo, S. Beucher, M. Bilodeau, E. Decencière, J.C. Klein, F. Meyer, F. Willot (Mathematical Morphology Center), C. Lantuéjoul (Geostatistics), Y. Bienvenu, G. Cailletaud, S. Forest, F. Grillon, M. Jeandin, F. N'Guyen, A. Pineau, J. Renard, J.L. Strudel, A. Thorel, J. P. Trottier (Centre des Matériaux P.M. Fourt). In addition to research, I had the pleasure to be involved in teaching courses on random media in Paris School of Mines and in other places, based on a large part of the content of this book. In this context, I am grateful to my former students, my fifty PhD students, and the numerous Postdocs, with whom luckily I was able to work over years on many topics with a strong support from industry. These topics are a fertile source of exciting problems in applied research. Physics of random media is a rich, active, and still promising domain of research and applications, which brought and still brings me great satisfaction. I had a long and fruitful cooperation with many partners, from both industrial and academic circles, to support students and to start new research projects. I am very grateful to them for their continuous support.

I am pleased to thank Springer International Publishing, and particularly L. Kunz and M. Peters, for the preparation of this edition.

Finally, I would never have been able to work on the material contained in this book without the relentless help of my wife Anne-Marie over so many years.

Symbols and Notations

Morphological operations

Minkowski addition of sets A and K:
$A \oplus K = \cup_{x \in A, y \in K}\{x+y\} = \cup_{y \in K} A_y = \cup_{x \in A} K_x$
Minkowski substraction of sets A and K: $A \ominus K = \cap_{y \in K} A_y = (A^c \oplus K)^c$
Dilation by a compact set $K : A \to A \oplus \check{K} = \{x, K_x \cap A \neq \emptyset\}$
Erosion by a compact set $K : A \to A \ominus \check{K} = \{x, K_x \subset A\}$
Opening by a compact set $K : A_K = A \ominus \check{K} \oplus K$
Closing by a compact set $K : A^K = A \oplus \check{K} \ominus K$

Indicator function of set A: $1_A(x) = 1$ if $x \in A$
usc (upper semi continuous) transformation
lsc (lower semi-continuous) transformation
Convex hull $C(A)$

Random sets

A, B: random closed sets (RACS)
A^c: complementary set of A
$\check{A} = \{-x, x \in A\}$: transposed set of A
A_i: component of a multi-component random set
$B(r)$: closed ball with radius r
K: compact set
E: topological space
$\mathcal{F}$, $\mathcal{G}$ and $\mathcal{K}$: closed, open and compact sets of E
$\mathcal{F}^K = \{F \in \mathcal{F};\ F \cap K = \emptyset,\ K \in \mathcal{K}\}$

$\mathcal{F}_G = \{F \in \mathcal{F};\, F \cap G \neq \emptyset,\, G \in \mathcal{G}\}$
Probability P
$G_K(s)$: generating function of the random variable $N(K)$
Choquet's capacity $T(K) = P\{K \cap A \neq \emptyset\} = P\{\mathcal{F}_K\} = 1 - P\{K \subset A^c\} = 1 - Q(K)$
$p = P\{x \in A\}$
$q = P\{x \in A^c\}$
Covariance $C(h) = P\{x \in A, x + h \in A\}$
Covariance $Q(h) = P\{x \in A^c, x + h \in A^c\}$
Covariance $C_{ij}(h) = P\{x \in A_i, x + h \in A_j\}$ for a multi-component random set
Three points Probability $Q(h_1, h_2) = P\{x \in A^c, x{+}h_1 \in A^c, x{+}h_2 \in A^c\}$
Segment l: $P(l) = P\{l \subset A\}$; $Q(l) = P\{l \subset A^c\}$
Hexagon $H(r)$: $P(H(r)) = P\{H(r) \subset A\}$; $Q(H(r) = P\{H(r) \subset A^c\}$
$R(x, A)$: distance between the point x and the set A

Autodual random sets: A and A^c have the same Choquet capacity

Measurements

$\mu(A)$: measure of A
μ_n Lebesgue measure in $\mathbb{R}^n$
Volume of A: $V(A)$
Integral of mean curvature of A: $M(A)$
Surface area of A: $S(A)$
Perimeter of A (in $\mathbb{R}^2$): $L(A)$
Specific connectivity number in $\mathbb{R}^2$: $N_A(A)$
Specific connectivity number in $\mathbb{R}^3$: $N_V(A) - G_V(A)$

Minkowski functionals of A: $W_i(A)$
Minkowski tensors of A: $W_\nu^{r,s}(A)$
Size distribution in number: $F(\lambda)$, in measure: $G(\lambda)$
Geodesic distance between $x \in A$ and $y \in A$: $d_A(x, y)$
Morphological tortuosity $\tau_A(x, y) = d_A(x, y) / \|x - y\|$

Random Functions

Random Function $Z(x)$
test function $g(x)$
$\Phi(E)$: set of functions from $E \to \overline{\mathbb{R}}$
upper semi continuous random functions (usc RF)
$\Phi_f \subset \Phi$: set of usc functions from $E \to \overline{\mathbb{R}}$
Choquet's capacity $T(g) = P\{x \in D_Z(g)\}; D_Z(g)^c = \{x, Z(x + y) < g(y), \forall y \in K\}$
lower semi continuous random functions (lsc RF)
$\Phi_g \subset \Phi$: set of lsc functions from $E \to \overline{\mathbb{R}}$
functional $P(g) = P\{x \in H_Z(g)\}; H_Z(g) = \{x, Z(x{+}y) \geq g(y), \forall y \in K\}$

subgraph Γ^{φ} of the function φ: $\Gamma^{\varphi} = \{x, z\}$, $x \in E$, $z \in \overline{\mathbb{R}}$, with $z \leq \varphi(x)$
overgraph Γ_{φ}: $\Gamma_{\varphi} = \{x, z\}$, $x \in E$, $z \in \overline{\mathbb{R}}$, with $z \geq \varphi(x)$
excursion set above level z: $A_Z(z) = \{x, Z(x) \geq z\}$
$\vee$: supremum; $Z_{\vee}(K) = \vee_{x \in K}\{Z(x)\}$
$\wedge$: infimum; $Z_{\wedge}(K) = \wedge_{x \in K}\{Z(x)\}$
dilation of Z by a function g (with $\check{g}(x) = g(-x)$): $Z \oplus \check{g}(x) = \vee_{y \in \mathbb{R}^n}\{Z(y) + g(y - x)\}$
erosion of Z by a function g (with $\check{g}(x) = g(-x)$): $Z \ominus \check{g}(x) = \wedge_{y \in \mathbb{R}^n}\{Z(y) - g(y - x)\}$
opening of Z by g: $\Psi_g(Z) = (Z \ominus \check{g}) \oplus g$
closing of Z by g: $\Psi^g(Z) = (Z \oplus \check{g}) \ominus g$
thresholding: $A_Z(z) = \{x, Z(x) \geq z\}$
spatial law: $F(x, z) = P\{Z(x_1) < z_1, ..., Z(x_m) < z_m\}$ with $x \in E^m$ and $z \in \overline{\mathbb{R}}^m$
spatial law: $T(x, z) = P\{Z(x_1) \geq z_1, ..., Z(x_m) \geq z_m\}$ with $x \in E^m$ and $z \in \overline{\mathbb{R}}^m$
$F(z)$, $G(z)$ distribution functions (with density, or pdf $f(z)$ and $g(z)$)
S: coefficient of variation of a distribution
$D^2[Z]$: variance of the random variable Z
Hermite polynomials $H_n(z)$
Bivariate distribution $F_{ij}(h, z_1, z_2) = P\{Z_i(x) < z_1, Z_j(x + h) < z_2\}$
Bivariate distribution $T_{ij}(h, z_1, z_2) = P\{Z_i(x) \geq z_1, Z_j(x + h) \geq z_2\}$
Bivariate distribution $T_2(h, z_1, z_2) = P\{Z(x) \geq z_1, Z(x + h) \geq z_2\}$
Covariance $C(x, x + h)$ and second order central correlation function $\overline{W}_2(x, x + h)$
Integral range A_n
RVE: Representative Volume Element
$\gamma_1(h)$, $\gamma_2(h)$: variograms of order 1 and 2
$g(h)$: transitive covariogram
Central correlation function of order m $\overline{W}_m(x)$, with $x \in E^m$

$\overline{Z}(V)$: average of $Z(x)$ in volume V
Mathematical expectation $E\{.\}$
$\Phi(u)$, $\Phi(u_1, u_2)$, $\Phi(u_1, u_2, ..., u_m)$: characteristic functions of $F(z)$, $F(z_1, z_2)$, $F(z_1, z_2, ..., z_m)$

Models

Random sets

BRS: Boolean random set model: RACS A with primary grain A'
Intensity θ
Primary grain A':
geometrical covariogram $K(h) = \overline{\mu}_n(A' \cap A'_{-h})$
normalized covariogram $r(h) = K(h)/K(0)$
$s(h_1, h_2) = \overline{\mu}_n(A' \cap A'_{-h_1} \cap A'_{-h_2})/K(0)$
IBRS: infinitesimal Boolean random set; time t; Intensity $\theta(t)$

DLRT: Dead Leaves tesselation: $N_i(t)$: specific number of intact grains; $\varphi_{A'}$, $\varphi_{A'_i}$: pdf of grains and of intact grains

STIT: random tessellation stable under iterations

Poisson varieties $V_k(\omega)$, with intensity $\theta(\omega)$

Probabilistic Texture segmentation: probabilistic distance between regions A_i and A_j: $P(A_i, A_j, d)$

Random functions (RF)

BRF: Boolean random function $Z(x)$

Gaussian RF

DLRF: Dead Leaves random function

TDLRF: Transparent Dead Leaves random function

IBRF: infinitesimal Boolean random function; time t; Intensity $\theta(t)$

MJF: sequential RF with Markovian jumps

SARF: Sequential alternate random function

$Z'_t(x)$: Primary random function, with subgraph $\Gamma^{Z'_t} = A'(t)$ and sections $A_{Z'_t}(z)$

$\varphi(Z)$: transformation of the RF Z by the anamorphosis φ

DRF: Dilution RF: $\Phi(U, X)$ and $\phi_t(U, X)$: multivariate characteristic functions of the RF $Z(x)$ and $Z'_t(x)$

$Z * \check{p}(x)$: convolution of the RF $Z(x)$ by a weight function $p(x)$

Reaction-Diffusion RF; coefficients of diffusion D_i

Change of scale in random media

Elasticity: stress σ, strain e, elasticity tensor $\mathbf{C}$; isotropic elasticity: bulk modulus K, shear modulus G, Young's modulus E, Poisson coefficient ν

Fluid flow: pressure gradient $\partial_i p$, fluid velocity u, kinematical viscosity μ; macroscopic flow rate Q, macroscopic pressure gradient $\partial_i P$, permeability K

Electrostatics: dielectric displacement D, electric field E, potential ϕ, permittivity ϵ, energy U

FE: Finite Elements

KUBC: Kinematic Uniform Boundary Conditions

SUBC: Static uniform Boundary Conditions

PBC: Periodic Boundary Conditions

FFT: Fast Fourier Transform

RVE: Representative Volume Element

Green's function in electrostatics $G(x, y)$

Operator $\Gamma(x, y)$ in electrostatics, with components $\Gamma_{ij}(x, y) = \frac{\partial^2}{\partial x_j \partial y_i} G(x, y)$

Hashin-Shrikman bounds H-S; upper H-S$^+$, lower H-S$^-$

$\varsigma_1(p)$ and $\eta_1(p)$: Milton functions for Beran-Molyneux-McCoy third order bounds

$P_n(u)$: Legendre polynomials

Fracture statistics: σ_R, fracture stress; $\Phi(\sigma)$, intensity of defects with critical stress $\sigma_c < \sigma$

Random fracture energy Γ; energy release rate G; fracture toughness G_c;
SVE: Statistical Volume Element in fracture
$\Phi(\sigma)$ phase field in fracture; Γ_{eff}: effective toughness

Contents