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Frédéric Barbaresco  
Frank Nielsen *Editors*

# Geometric Structures of Statistical Physics, Information Geometry, and Learning

SPIGL'20, Les Houches, France,  
July 27–31

 Springer

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# Preface

## **Geometric Structures of statistical Physics, Information Geometry, and Learning**

*Ecole de Physique des Houches SPIGL'20 Summer Week*

**SPRINGER Proceedings in Mathematics & Statistics, 2021**

### **Subject**

This book is proceedings of Les Houches Summer Week SPIGL'20 (Joint Structures and Common Foundation of **Statistical Physics, Information Geometry and Inference for Learning**) organized from July 27–31, 2020, at L'Ecole de Physique des Houches:

Website <https://franknielsen.github.io/SPIG-LesHouches2020/>

Videos: <https://www.youtube.com/playlist?list=PLo9ufcrEqwWExTBPgQPJwAJhoUChMbROr>

The conference SPIGL'20 has developed the following topics:

### *Geometric Structures of Statistical Physics and Information*

- Statistical mechanics and geometric mechanics
- Thermodynamics, symplectic and contact geometries
- Lie groups thermodynamics
- Relativistic and continuous media thermodynamics
- Symplectic integrators



## Scientific Rational

In the middle of the last century, Léon Brillouin in “The Science and The Theory of Information” or André Blanc-Lapierre in “Statistical Mechanics” forged the first links between the theory of information and statistical physics as precursors.

In the context of artificial intelligence, machine learning algorithms use more and more methodological tools coming from the physics or the statistical mechanics. The laws and principles that underpin this physics can shed new light on the conceptual basis of artificial intelligence. Thus, the principles of maximum entropy, minimum of free energy, Gibbs–Duhem’s thermodynamic potentials and the generalization of François Massieu’s notions of characteristic functions enrich the variational formalism of machine learning. Conversely, the pitfalls encountered by artificial intelligence to extend its application domains question the foundations of statistical physics, such as the construction of stochastic gradient in large dimension, the generalization of the notions of Gibbs densities in spaces of more elaborate representation like data on homogeneous differential or symplectic manifolds, Lie groups, graphs, and tensors.

Sophisticated statistical models were introduced very early to deal with unsupervised learning tasks related to Ising–Potts models (the Ising–Potts model defines the interaction of spins arranged on a graph) of statistical physics and more generally the Markov fields. The Ising models are associated with the theory of mean fields (study of systems with complex interactions through simplified models in which the action of the complete network on an actor is summarized by a single mean interaction in the sense of the mean field).

The porosity between the two disciplines has been established since the birth of artificial intelligence with the use of Boltzmann machines and the problem of robust methods for calculating partition function. More recently, gradient algorithms for neural network learning use large-scale robust extensions of the natural gradient of Fisher-based information geometry (to ensure reparameterization invariance), and stochastic gradient based on the Langevin equation (to ensure regularization), or their coupling called “natural Langevin dynamics”.

Concomitantly, during the last fifty years, statistical physics has been the object of new geometrical formalizations (contact or symplectic geometry, ...) to try to give a new covariant formalization to the thermodynamics of dynamic systems. We can mention the extension of the symplectic models of geometric mechanics to statistical mechanics, or other developments such as random mechanics, geometric mechanics in its stochastic version, Lie groups thermodynamics, and geometric modeling of phase transition phenomena.

Finally, we refer to computational statistical physics, which uses efficient numerical methods for large-scale sampling and multimodal probability measurements (sampling of Boltzmann–Gibbs measurements and calculations of free energy, metastable dynamics and rare events, ...) and the study of geometric integrators (Hamiltonian dynamics, symplectic integrators, ...) with good properties of covariances and stability (use of symmetries, preservation of invariants, ...).



Machine learning inference processes are just beginning to adapt these new integration schemes and their remarkable stability properties to increasingly abstract data representation spaces.

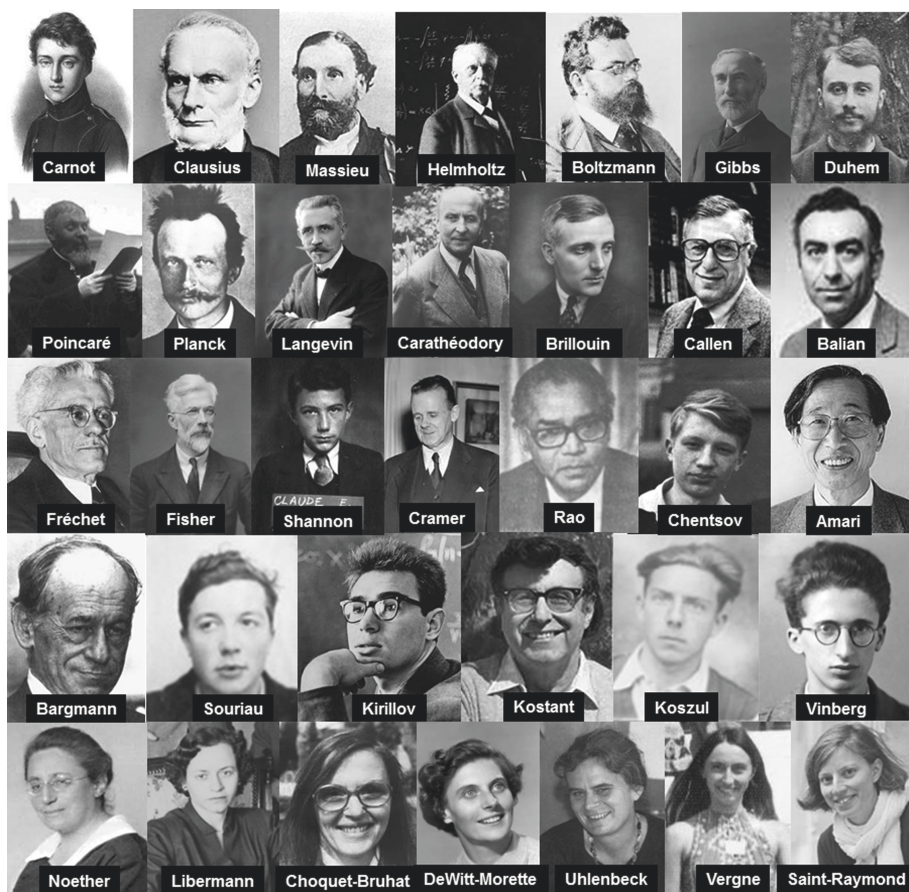
Artificial intelligence currently uses only a very limited portion of the conceptual and methodological tools of statistical physics. The purpose of this conference is to encourage constructive dialog around a common foundation, to allow the establishment of new principles and laws governing the two disciplines in a unified approach. However, it is also about exploring new chemins de traverse.

**Joint Structures and Common Foundations of Statistical Physics, Information Geometry and Inference for Learning  
(SPIGL July 27-31th 2020, Les Houches, France)**



<u>Onsite participants:</u>	<u>Online participants:</u>
Eric Moulines	Elena Celledoni
Jean-Pierre Francoise	Francoisco Chinesta
Jean-Claude Zambini	François Gay-Balmaz
Gery de Saucy	Manuel de León
Frédéric Barbaresco	Steve Huntsman
Goffredo Chirco	Bernhard Maschke
Luigi Malago	Michel Nguffo Boyom
Vân Lê	Frank Nielsen
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Main contributors in thermodynamics, statistical physics, information geometry and Lie group representation theory:



April 2021

Frédéric Barbaresco  
Frank Nielsen

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