Springer Proceedings in Mathematics & Statistics

Stefania Ugolini · Marco Fuhrman ·
Elisa Mastrogiacomo · Paola Morando ·
Barbara Rüdiger *Editors*

Geometry and Invariance in Stochastic Dynamics

Verona, Italy, March 25–29, 2019

Springer

Springer Proceedings in Mathematics & Statistics

Volume 378

This book series features volumes composed of selected contributions from workshops and conferences in all areas of current research in mathematics and statistics, including data science, operations research and optimization. In addition to an overall evaluation of the interest, scientific quality, and timeliness of each proposal at the hands of the publisher, individual contributions are all refereed to the high quality standards of leading journals in the field. Thus, this series provides the research community with well-edited, authoritative reports on developments in the most exciting areas of mathematical and statistical research today.

More information about this series at https://link.springer.com/bookseries/10533

Stefania Ugolini · Marco Fuhrman ·
Elisa Mastrogiacomo · Paola Morando ·
Barbara Rüdiger
Editors

Geometry and Invariance in Stochastic Dynamics

Verona, Italy, March 25–29, 2019

Springer

Editors
Stefania Ugolini
Department of Mathematics
Università degli Studi di Milano
Milan, Italy

Elisa Mastrogiacomo
Department of Economics
Università degli Studi dell'Insubria
Varese, Italy

Barbara Rüdiger
Mathematic and Informatic Department
Bergische Universität Wuppertal
Wuppertal, Germany

Marco Fuhrman
Department of Mathematics
Università degli Studi di Milano
Milan, Italy

Paola Morando
Department of Agricultural
and Environmental Sciences
Università degli Studi di Milano
Milan, Italy

ISSN 2194-1009 ISSN 2194-1017 (electronic)
Springer Proceedings in Mathematics & Statistics
ISBN 978-3-030-87431-5 ISBN 978-3-030-87432-2 (eBook)
https://doi.org/10.1007/978-3-030-87432-2

Mathematics Subject Classification: 60HXX, 60H15, 34C15, 35B06, 37HXX

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Proceedings CONFERENCE VERONA 2019 dedicated to Albeverio's 80th birthday

Geometry and Invariance in Stochastic Dynamics

Sala Capitolare, Chiostro di San Giorgio in Braida, 25–29 March 2019

Preface

Geometry and Invariance in Stochastic Dynamics

The study of symmetries and invariance properties of ordinary and partial differential equations (ODE and PDE resp.) is a classical and well-developed area of research and provides a powerful tool for computing some explicit solutions to the equations and analyzing their qualitative behavior.

In the last decades, the fruitful notions of symmetry and invariance have been extended beyond classical mechanical systems. In fact, the development of geometric mechanics allowed the generalization of these notions to more complex (finite and infinite dimensional) systems arising in many different areas of physics.

It is well known that variational principles, and their associated Hamiltonian formulations, provide a natural framework for both classical and modern physics, such as general relativity and quantum mechanics, and constitute one of the most useful tools in mathematical physics. In particular, the physics of the XXth century was deeply influenced by the fundamental theorem of Noether, which associates symmetries of a classical dynamical system with its invariants, i.e., with quantities that remain constant during the time evolution of the system.

The modern theory of symmetry for ODEs and PDEs, due to S. Lie, is based on the extension of the original concept of discrete group introduced by E. Galois to continuous groups of transformations. Nowadays, Lie's symmetry theory is widely applied both to ODEs and PDEs in order to reduce the original system to a simpler form, exploiting symmetry-adapted coordinates.

From a numerical point of view, the theory of geometric numerical integration for ODEs focuses on the preservation of continuous geometric structures under time discretization. It provides a powerful alternative to standard discretization methods with a significant impact on the theoretical and practical aspects of modern computational mathematics.

Another research area involving symmetry and invariance arises from the interaction of quantum theory and probability. Indeed, since the first quarter of last century, the development of the theory of general relativity enhanced the traditional

links between classical deterministic mechanics and analysis and set off emerging research fields between algebra and geometry. At the same time, quantum mechanics connected analysis and probability and in the second half of last century, these interactions spread to quantum field theory and to the study of singular partial differential equations.

Despite the big achievements obtained in the deterministic setting, the importance of the study of invariance properties and geometric structure of finite- or infinite-dimensional *stochastic differential equations* (SDEs and SPDEs resp.) has been overlooked for a long time and a systematic generalization of the deterministic results to the stochastic framework is much needed from both practical and theoretical point of view.

The purpose of this book is to collect contributions in this direction and to provide an overview that can inspire further researches aiming at generalizing to the stochastic framework results involving geometric structures and invariance properties of deterministic ODEs and PDEs. For this reasons, the papers included in the volume range from theoretical probability to the study of geometric and algebraic structures, offering an extraordinary opportunity to approach this promising research field from different perspectives.

In the following, without claiming to be complete, we try to outline the main research topics that are tackled in the book.

From a geometric point of view, the generalization of geometric mechanics, which is essentially based on group-invariant variational principles, to (Stratonovich) stochastic setting gave rise to the new research area called *stochastic geometric mechanics* (a beautiful introduction to the subject can be found in the Springer Volume titled Stochastic Geometric Mechanics, 2017). Important achievements of this stochastic extension are the variational formulation of SDEs and the Euler-Poincarè reduction of stochastic infinite-dimensional variational systems in stochastic fluid-dynamics. In particular, recent theoretical results in this framework turned out to be extremely useful in order to obtain advanced numerical analysis techniques.

On the other hand, a direct (but non-trivial) extension of Lie symmetries approach to the stochastic setting can be successfully exploited in order to determine explicit solutions to SDEs, to reduce and reconstruct symmetric SDEs as well as to find finite-dimensional solutions to SPDEs.

Moreover, recent advances unveil the central role played by algebraic structures such as pre-Lie and post-Lie algebras, and their enveloping algebras which permit to join B-series and Lie-series into Munthe-Kaas' Lie-Butcher series on manifolds. These structures turn out to be associated with Euclidean geometry and with homogeneous manifolds and Lie groups, and recent results in this framework provide an interesting extension of the theory of Lie group integration to nonlinear SDEs. Moreover, the Lie and Hopf algebraic setting underlying Lie group integration was recently adapted to Lyons' theory of rough paths, extending the notion of rough differential equations to homogeneous spaces. In this area advanced combinatorial methods have been successfully applied.

Furthermore, the interest in the study of singular PDEs, renormalization theory for quantum fields and critical phenomena in statistical mechanics gave a big boost

to new researches on SPDEs that experienced striking new developments in recent years.

On the other side, the investigation of *invariance properties* for stochastic processes provides an interesting and well-established research topic in theoretical probability.

When stochastic processes take their values in manifolds, the standard probabilistic tools have to interact with differential geometry techniques, originating a research field called *stochastic differential geometry*. Since Lie groups combine the algebraic structure of a group with the geometric notion of differential manifold, an interesting challenge is the study of diffusions, Markov processes, and Lévy processes on Lie groups. In particular, the analysis of the invariance properties of this kind of processes under the action of the group gives promising results in the study of SDEs driven by Lévy processes (also in the jump case) as well as in their characterization in terms of stochastic variational principles. Moreover, the investigation of a stochastic process by means of its invariance under random transformations provides useful characterizations of the process itself.

Since many equations are not perfectly symmetric, another interesting theoretical emerging area is the perturbations of symmetric or Hamiltonian systems. In this setting, the symmetries of a diffusion equation can also be exploited in order to obtain precise analytical properties of the related semigroup. These results have important applications in the case of symmetric spaces with invariant Riemannian structures.

Finally, the knowledge of some closed formula is also crucial in many applications of stochastic processes, since it permits to develop faster and cheaper numerical algorithms for the simulation of the process or to evaluate interesting quantities related to it, such as martingales which can be individuated through stochastic Noether theorem as counterparts of the deterministic conserved quantities.

The Conference, titled *Random Transformations and Invariance in Stochastic Dynamics* (25–29 March 2019), held in the cloister of San Giorgio in Braida in Verona (Italy) and was dedicated to Sergio Albeverio for his 80th birthday.

In connection with the conference two volumes are being published. A first volume consists of contributions directly related to the extraordinarily rich and exciting human and scientific adventure of Sergio Albeverio; the present second volume, as recalled above, contains the main research lines on geometry and invariance in stochastic dynamics.

The editors of the present volume thank the authors who, besides actively participating to the conference, accepted the invitation to write up their contributions. We also thank other lecturers at the conference who greatly collaborated to the success of the event and all the participants, for their presence and their active contribute to create a really agreeable and inspiring atmosphere.

Many thanks to Marina Reizakis of Springer-Verlag, for accepting our invitation to attend the Conference and to put on display books related to the conference. Her competent and stimulating advice during the preparation of these proceedings was greatly appreciated. We also thank Banu Dhayalan of Springer-Verlag for their technical support in the preparation of the printing process.

Special thanks to the Rector of San Giorgio in Braida, Don Giorgio Marchesi, for the permission to enjoy the Renaissance chapter house and the beautiful cloister on the banks of Adige river who constituted the venue of the Conference.

The editors are also grateful to the following institutions for the financial support: University of Milano (Transition grant to S. Ugolini), GNAMPA (INDAM), University of Trento, Linneaus University, University of Wuppertal, and University of Insubria.

Milan, Italy — Stefania Ugolini
Milan, Italy — Marco Fuhrman
Varese, Italy — Elisa Mastrogiacomo
Milan, Italy — Paola Morando
Wuppertal, Germany — Barbara Rüdiger

Random Transformations and Invariance in Stochastic Dynamics

Sala Capitolare, Chiostro di San Giorgio in Braida, Verona (Italy), 25–29 March 2019

Conference in Verona: List of Participants

Sergio Albeverio, University of Bonn, HCM, Germany, albeverio(at)iam-bonn.de
Ana Bela Cruzeiro, University of Lisboa, Portugal, ana.cruzeiro(at)tecnico.ulisboa.pt
Francesco De Vecchi, University of Bonn, HCM, Germany, fdevecchi(at)uni-bonn.de
David Elworthy, University of Warwick, England, K. D. Elworthy(at)warwick.ac.uk
Darryl Holm, Imperial College, England, d.holm(at)imperial.ac.uk
Paul Lescot, Université de Rouen, France, paul.lescot(at)univ-rouen.fr
Xue-Mei Li, Imperial College, England, xue-mei.li(at)imperial.ac.uk
Juan Pablo Ortega, Universitat Sankt Gallen, Switzerland, juan-pablo.ortega(at)unisg.ch
Frédric Patras, Université Côte d'Azur, France, patras(at)unice.fr
Nicolas Privault, Nanyang Technological University, Singapore, nprivaul(at)ntu.edu.sg
Tudor Ratiu, University of California, USA, tratiu(at)scsc.edu
Francesco Russo, Institut Polytecnique de Paris, ENSTA, France, francesco.russo(at)ensta-paris.fr
Laurene Valade, Université de Rouen, France, laurene.valade(at)etu.univ-rouen.fr
Jean Claude Zambrini, University of Lisboa, Portugal, jczambrini(at)fc.ul.pt
Michael Rochner, University of Bielefeld, Germany, roechner(at)math.uni-bielefeld.de
Philippe Blanchard, University of Bielefeld, Germany, blanchard(at)physik.uni-bielefeld.de
Minoru Yoshida, Kanagawa University, Japan, washizuminoru(at)Hotmail.com

Claudio Cacciapuoti, University of Insubria, Italy,
Claudio.cacciapuoti(at)uninsubria.it
Fabio Cipriani, Politecnico di Milano, Italy, fabio.cipriani(at)polimi.it
Alexei Daletskii, University of York, England, alex.daletskii(at)york.ac.uk
Gianfausto Dell'Antonio, SISSA (Scuola Internazionale Superiore di Studi Avanzati), Italy, gianfa(at)sissa.it
Luca Di Persio, University of Verona, Italy, luca.dipersio(at)univr.it
Benedetta Ferrario, University of Pavia, Italy, benedetta.ferrario(at)unipv.it
Rodolfo Figari, University of Napoli, Italy, figari(at)na.infn.it
Francesco Guerra, University of Roma ''La Sapienza", Italy,
francesco.guerra(at)roma1.infn.it
Nadia Robotti, University of Genova, Italy, robotti(at)fisica.unige.it
Hanno Gottschalk, University of Wuppertal, Germany, hanno.gottschalk(at)uni-wuppertal.de
Yuri Kondratiev, University of Bielefeld, Germany, kondrat(at)math.uni-bielefeld.de
Pavel Kurasov, University of Stockholm, Norvey, kurasov(at)math.su.se
Vidadhar Mandrekar, University of Michigan, USA, mandrekar(at)stt.msu.edu
Tom Lindstrom, University of Oslo, lindstro(at)math.uio.no
Carlo Marinelli, University College London, England, c.marinelli(at)ucl.ac.uk
Danilo Merlini, CERFIM, Switzerland, merlini(at)cerfim.ch
Andrea Posilicano, University of Insubria, Italy, andrea.posilicano(at)uninsubria.it
Alessandro Teta, University of Roma ''La Sapienza", Italy, teta(at)mat.uniroma1.it
Luciano Tubaro, University of Trento, Italy, luciano.tubaro(at)unitn.it
Andrea Romano, MatemUpper Association, andrea.23.romano(at)gmail.com
Marco Tarsia, University of Uninsubria, Italy, masia1(at)uninsubria.it
Francesco Cordoni, University of Verona, Italy,
francescogiuseppe.cordoni(at)univr.it
Walter Moretti, University of Trento, Italy, valter.moretti(at)unitn.it
Giulia Basti, GSSI L'Aquila, Italy, giulia.basti(at)gssi.it
Luigi Galgani, University of Milano, Italy, luigi.galgani(at)unimi.it
Diego Noja, University of Milano Bicocca, Italy, diego.noja(at)unimib.it

Conference organizers:

Stefania Ugolini, University of Milano, Italy, stefania.ugolini(at)unimi.it
Marco Fuhrman, University of Milano, Italy, marco.fuhrman(at)unimi.it
Astrid Hilbert, Linnaeus University, astrid.hilbert(at)lnu.se
Elisa Mastrogiacomo, University of Insubria, Italy,
elisa.mastrogiacomo(at)uninsubria.it
Sonia Mazzucchi, University of Trento, Italy, sonia.mazzucchi(at)unitn.it
Paola Morando, University of Milano, Italy, paola.morando(at)unimi.it
Barbara Rüdiger, University of Wuppertal, Germany, ruediger(at)uni-wuppertal.de

Contents