

ICIAM 2019 SEMA SIMAI Springer Series 13

Sebastià Xambó-Descamps *Ed.*

Systems, Patterns and Data Engineering with Geometric Calculi



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The sub-series is managed by an independent Editorial Board, and will include peer-reviewed content only, including the Invited Speakers volume as well as books resulting from mini-symposia and collateral workshops.

The series is aimed at providing useful reference material to academic and researchers at an international level.

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Preface

The genesis of this collection is the homonymous mini-symposium (MS) held on July 16, 2019, in the **International Congress of Industrial and Applied Mathematics (ICIAM-2019)** organized by the **Spanish Society of Applied Mathematics (SEMA)** at the **University of Valencia** (July 15–19, 2019). Its initial idea, however, was born out of discussions I held with Carlile LAVOR, Pablo COLAPINTO and Srđan LAZENDIĆ on the occasion of the 7th Conference on *Applied Geometric Algebras in Computer Science and Engineering (AGACSE 2018)*, a satellite of ICIAM-2019 held at the University of Campinas, Brazil, July 23rd to 27th, 2018.

The goal of the mini-symposium was envisaged to overview the basic ideas of geometric algebra/calculus, to report on state-of-the-art applications showcasing its advantages, and to explore the bearing of the formalisms in novel contexts, with a particular view to automatic learning. The idea was given further considerations, within the constraints of the ICIAM-2019 MS, and finally the outcome was the following program:

1. *Geometric calculus techniques in science and engineering* (Sebastià XAMBÓ-DESCAMPS, Chair)
2. *Bringing new perspectives to robotics and computer vision* (Isiah ZAPLANA)
3. *Geometric algebra and distance geometry* (Carlile LAVOR)
4. *Embedded Coprocessors for Native Execution of Geometric Algebra Operations* (Salvatore VITABILE)
5. *Hypercomplex algebras for art investigation* (Srđan LAZENDIĆ)
6. *Conformal Geometric Algebra for Medical Imaging* (Salvatore VITABILE)
7. *Geometric bio-inspired deep learning* (Eduardo Ulises MOYA)
8. *Geometric Calculus meets Deep Learning* (Sebastià XAMBÓ-DESCAMPS)

E. U. MOYA had send a paper to the AGACSE 2018 Conference, and although it turned out that he could not travel to Campinas (his work was presented there by Eduardo BAYRO-CORROCHANO, his PhD advisor), he expected to attend ICIAM-2019. In contrast, Pablo COLAPINTO attended AGACSE 2018, and there he

agreed to deliver a talk at the MS on *The New Geometry of Computer Aided Design* if he could manage to join the MS, but at the end he could not.

The collection of papers in this volume differs from the program in several respects. The talk number 5 did not end in a paper, nor did the tutorial 1. The talks 4 and 6 were merged into a single paper. In addition to the five remaining contributions (perhaps with additional authors and variations in the titles), we invited three additional authors: Leo DORST, Pablo COLAPINTO, and Leandro A. F. FERNANDES. The final contents are as follows:

1. *New Perspectives on Robotics with Geometric Calculus* (Isiah ZAPLANA)
2. *Recent advances on oriented conformal geometric algebra applied to molecular distance geometry* (Carlile LAVOR and Rafael ALVES)
3. *Geometric Calculus Applications to Medical Imaging: Status and Perspectives* (Silvia FRANCHINI and Salvatore VITABILE)
4. *Optimal Combination of Orientation Measurements Under Angle, Axis and Chord Metrics* (Leo DORST)
5. *Space-Bending Lattices through Conformal Transformation of Principal Contact Elements* (Pablo COLAPINTO)
6. *Exploring Lazy Evaluation and Compile-Time Simplifications for Efficient Geometric Algebra Computations* (Leandro A. F. FERNANDES)
7. *A Quaternion Deterministic Monogenic CNN Layer for Contrast Invariance* (Eduardo U. MOYA, S. XAMBÓ-DESCAMPS, Sebastián SALAZAR COLORES, Abraham SÁNCHEZ, Ulises CORTÉS)
8. *Geometric Calculus and Deep Learning—An Overview* (S. XAMBÓ-DESCAMPS and Eduardo U. MOYA)

The main idea of this collection is well aligned with the core purposes of the MS. The first three contributions, which correspond to lectures at the MS, offer perspectives on recent advances in the application GC in the areas of robotics, molecular geometry, and medical imaging. The next three, especially invited, hone the expressiveness of GC in orientation measurements under different metrics, the treatment of contact elements, and the investigation of efficient computational methodologies. The last two, which also correspond to lectures at the MS, deal with two aspects of deep learning: a presentation of a concrete quaternionic convolutional neural network layer for image classification that features contrast invariance and a general overview of automatic learning aimed at steering the development of neural networks whose units process elements of a suitable algebra, as, for instance, a geometric algebra.

It is a pleasant duty to thank the organizers of ICIAM-2019, and in particular the organizers of the mini-symposia, the speakers at our MS, and the authors of the papers collected in this volume.

Contents

New Perspectives on Robotics with Geometric Calculus	1
Isiah Zaplana	
Recent Advances on Oriented Conformal Geometric Algebra Applied to Molecular Distance Geometry	19
Carlile Lavor and Rafael Alves	
Geometric Calculus Applications to Medical Imaging: Status and Perspectives	31
Silvia Franchini and Salvatore Vitabile	
Optimal Combination of Orientation Measurements Under Angle, Axis and Chord Metrics	47
Leo Dorst	
Space-Bending Lattices: Parameterization and Rationalization of Cyclidic Volumes Through Conformal Transformation of Principal Contact Elements	89
Pablo Colapinto	
Exploring Lazy Evaluation and Compile-Time Simplifications for Efficient Geometric Algebra Computations	111
Leandro A. F. Fernandes	
A Quaternion Deterministic Monogenic CNN Layer for Contrast Invariance	133
Eduardo Ulises Moya-Sánchez, Sebastià Xambó-Descamps, Sebastián Salazar Colores, Abraham Sánchez Pérez, and Ulises Cortés	
Geometric Calculi and Automatic Learning <i>An Outline</i>	153
Sebastià Xambó-Descamps and Eduardo Ulises Moya	
Author Index	179