

SpringerBriefs in Computer Science

Robert Kudelić

# Feedback Arc Set

A History of the Problem  
and Algorithms

# SpringerBriefs in Computer Science

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# Foreword

The feedback arc set problem is a one of the quintessential problems of algorithmics and, more generally, of computer science. The problem is easily stated: for a given directed graph  $G = (V, E)$ , find the smallest subset  $E' \subset E$  such that  $G' = (V, E \setminus E')$  is acyclic. The problem is well-known and has been investigated by the best minds in computer science. It arises as a fundamental issue in many problems in industry, from circuit design to logistics. Yet, the problem has proven very difficult to solve.

Over the last fifty years, methods to solve the feedback arc set problem have spanned the length and breadth of the techniques of computer science: paradigms of graph algorithms, well-designed data structures, optimization via integer programming, heuristics with proven performance guarantees, “soft computing” methods such as ant algorithms, and stochastic methods such as Monte-Carlo algorithms.

The author Robert Kudešić has published papers on several disparate approaches to the feedback arc set problem. His book takes the reader chronologically through the feedback arc set literature, and as such it provides a historical compendium of techniques in computer science.

Sydney, NSW, Australia  
January 2022

Peter Eades

# Preface

As is true for any subject in science, there comes a time when one needs to look back, and take a gaze upon work done thus far. And if we are to choose such a time for a classical computer science problem known as feedback arc set (FAS), and more than half a century has passed since its fledgling days, this is a good time to work on that gaze. A general introduction to the problem will be presented, hardness and importance will be argued. Span will be given from the origin and the name, through problem definition and various versions, all the way to the dual version and a link with the feedback vertex set and the forcing problem—practical relevance will be covered as well.

By building on that foundation, a review of the algorithms will be given, with special emphasis on FAS. The goal of this review is to give both depth, through covering algorithms, and breath, through spanning lifetime of the FAS problem. It is the intention that, through this book, any interested party will be able to quickly find relevant information, fostering both learning and research.<sup>1</sup>

There are a number of people that were of help to me during preparation of this book, and therefore a few words of gratitude are in order. I would like to thank my college for suggesting that the material which is found in this work, a work in progress at that time, would be a good fit for a book—the idea quickly grew in my mind, and the publisher was soon found.

I would also like to express my sincere gratitude to Peter Eades, well known for his expertise in graph problems, for being so kind and taking the time to write the foreword to this book.

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<sup>1</sup> Online supplementary material is available at: <https://cs.foi.hr/fas/book/>.

A special appreciation would go to my father for enabling my academic career. Gratitude is in order for a colleague of mine, Nikola Ivković, who was very helpful in acquiring a number of papers for me. And I can't forget Wayne Wheeler, for the support and for being such a nice person to talk too, it was wonderful.

Varaždin, Croatia  
April 2022

Robert Kudelić



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