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# Control and Estimation of Systems with Input/Output Delays

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# Preface

Time delay systems exist in many engineering fields such as transportation, communication, process engineering and more recently networked control systems. In recent years, time delay systems have attracted recurring interests from research community. Much of the research work has been focused on stability analysis and stabilization of time delay systems using the so-called Lyapunov-Krasovskii functionals and linear matrix inequality (LMI) approach. While the LMI approach does provide an efficient tool for handling systems with delays in state and/or inputs, the LMI based results are mostly only sufficient and only numerical solutions are available.

For systems with known single input delay, there have been rather elegant analytical solutions to various problems such as optimal tracking, linear quadratic regulation and $H_\infty$ control. We note that discrete-time systems with delays can usually be converted into delay free systems via system augmentation, however, the augmentation approach leads to much higher computational costs, especially for systems of higher state dimension and large delays. For continuous-time systems, time delay problems can in principle be treated by the infinite-dimensional system theory which, however, leads to solutions in terms of Riccati type partial differential equations or operator Riccati equations which are difficult to understand and compute. Some attempts have been made in recent years to derive explicit and efficient solutions for systems with input/output (i/o) delays. These include the study on the $H_\infty$ control of systems with multiple input delays based on the stable eigenspace of a Hamlitonian matrix [46]. It is worth noting that checking the existence of the stable eigenspace and finding the minimal root of the transcendent equation required for the controller design may be computationally expensive. Another approach is to split a multiple delay problem into a nested sequence of elementary problems which are then solved based on J-spectral factorizations [62].

In this monograph, our aim is to present simple analytical solutions to control and estimation problems for systems with multiple i/o delays via elementary tools such as projections. We propose a re-organized innovation analysis approach which allows us to convert many complicated delay problems into delay free ones. In particular, for linear quadratic regulation of systems with multiple input delays, the approach enables us to establish a duality between the LQR problem and a smoothing problem for a delay free system. The duality contains the well known duality between the LQR of a delay free system and Kalman filtering as a special case and allows us to derive an analytical solution via simple projections. We also consider the dual problem, i.e. the Kalman filtering for systems with multiple delayed measurements. Again, the re-organized innovation analysis turns out to be a powerful tool in deriving an estimator. A separation principle will be established for the linear quadratic Gaussian control of systems with multiple input and output delays. The re-organized innovation approach is further applied to solve the least mean square error estimation for systems with multiple state and measurement delays and the $H_\infty$ control and estimation problems for systems with i/o delays in this monograph.

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# Symbols and Acronyms

| | |
|---|---|
| i/o: | input/output. |
| LQG: | linear quadratic Gaussian. |
| LQR: | linear quadratic regulation. |
| PDE: | partial differential equation. |
| RDE: | Riccati difference (differential) equation. |
| $col\{X_1, \cdots, X_n\}$: | the column vector formed by vectors $X_1, \cdots, X_n$. |
| $R^n$: | $n$-dimensional real Euclidean space. |
| $R^{n\times m}$: | set of $n \times m$ real matrices. |
| $I_n$: | $n \times n$ identity matrix. |
| diag$\{A_1, A_2, \cdots, A_n\}$: | block diagonal matrix with $A_j$( not necessarily square) on the diagonal. |
| $X'$: | transpose of matrix $X$. |
| $P \geq 0$: | symmetric positive semidefinite matrix $P \in R^{n\times n}$. |
| $P > 0$: | symmetric positive definite matrix $P \in R^{n\times n}$. |
| $P^{-1}$: | the inverse of the matrix $P$. |
| $\langle X, Y\rangle$ : | inner product of vectors $X$ and $Y$. |
| $\mathcal{E}$: | mathematical expectation. |
| $\mathcal{L}\{y_1, \cdots, y_n\}$: | linear space spanned by $y_1, \cdots, y_n$. |
| $\triangleq$: | definition. |
| $dim(x)$: | the dimension of the vector $x$. |
| $Proj$: | projection |

$MD$: the total number of Multiplications and Divisions.

$\delta_{ij}$: $\delta_{ij} = \begin{cases} 1, \; i = j, \\ 0, \; i \neq j. \end{cases}$

$\|e\|_2$ : $\ell_2$-norm of a discrete-time signal $\{e(i)\}$, i.e., $\sqrt{\sum_{i=0}^{\infty} \|e(i)\|^2}$.

$\ell_2[0, N]$: space of square summable vector sequences on $[0, N]$ with values on $R^n$.

$\mathcal{L}_2[0, t_f]$: space of square integrable vector functions on $[0, t_f]$ with values on $R^n$.

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