Séminaires IRIA

analyse et contrôle des systèmes

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PRÉFACE

Ce volume regroupe des articles correspondants aux séminaires d'Analyse Numérique et d'Automatique organisés à l'IRIA-LABORIA en 1973 - les textes reproduits sont fournis par les conférenciers à l'issue de leurs séminaires.

Leur lecture permettra de faire une synthèse des préoccupations récentes d'équipes de recherche provenant d'horizons très variés, tant pour des raisons géographiques, puisque des pays comme l'Argentine et la Pologne sont représentés, que par la diversité des sujets abordés. Cet ouvrage pourra, grâce à une diffusion rapide, être un outil de travail utile à tous les chercheurs en Mathématiques Appliquées.

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NATURAL GEOMETRY OF SURFACES WITH SPECIFIC REFERENCE TO THE MATRIX DISPLACEMENT ANALYSIS OF SHELLS

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1. Introduction

It is possibly a truism to assert that the static and dynamic analysis of shells of arbitrary form can nowadays be accomplished best by the finite element or matrix methods. Whilst purely static investigation may no doubt be carried out with some advantage - at least in the linear or small displacement field - using a so-called mixed model, it is equally evident that an extension allowing for dynamic and large displacement effects calls for a kinematic idealisation of the elements (i.e. the matrix displacement method). Thus, it appears that for most applications and certainly from the point of view of a general purpose program a kinematic modelling is, as usual, much to be preferred. To any engineer it is also evident that an efficient idealisation of an arbitrary shell is best achieved with 'triangular' surface elements, having an arbitrary varying curvature. These enjoy another significant advantage: they are the only type of shell elements which permit a mapping procedure for the middle surface and the displacements with complete polynomials. The marked theoretical and practical advantages of complete polynomials, e.g. retention of isotropy, have been discussed on a number of occasions; see e.g. [3] .

A prerequisite of interelement kinematic compatibility - which must include displacements and rotations about the tangent to the boundary - is secured by prescribing the same mapping or interpolation technique to the components of the position vector (for the undeformed geometry) and of the displacement vector. Ficticious folds at the interelement junctions prior or subsequent to the deformation, are thereby avoided. The selected interpolation procedure is seen to satisfy also the second essential condition for a correct kinematic idealisation - namely the existence of true rigid body modes which do not give rise to strains. This means on the other hand that overall equilibrium of the element is satisfied exactly.

The lowest order that may be used is easily seen to be five and this should suffice for most applications. At the same time a slight blemish is inevitably associated with this procedure, since it extends the interelement kinematic compatibility condition to the second derivatives at the vertices, which do not appear in the corresponding variational statement. However, this overcompatibility does not seem to cause, in any problem considered to-date, any noticeable increase of the stiffness. The theory of these elements, known as SHEBA, has been developed in [1,11]. A review of its practical application to a number of examples including the simulation of singularities is given in [9].

Already in the first conceptual steps leading to the development of the SHEBA element it became apparent that considerable benefit is accrued in the formation of the theory if instead of the traditional cartesian convention for the stress we introduce the natural concepts initiated in [8]. Thus, instead of the usual definition of cartesian direct and shear stresses we specify the stress state solely by direct measures in three different directions. In an elementary membrane element TRIM 3 these directions are taken parallel to the three sides. Our new specification of stress applies in the present case not only to the membrane but also the bending contributions. This natural decomposition leads to a more elegant and symmetrical theory and is ideally suited for an extension into the non-linear domain for large displacements [2].

However, the full measure of the advantages associated with a so-called natural point of view becomes only evident if they are extended into the geometrical aspects of the theory. To appreciate this let us recapitulate the essential basis of the mapping technique. In particular, the middle surface of the arbitrarily curved SHEBA elements is generated through mapping of a triangle in the so-called parameter plane via the afore-mentioned fifth order complete polynomials. As we have advocated on a number of occasions a point in a plane may be fixed symmetrically with respect to a triangle if we specify it by its so-called homogeneous triangular co-ordinates ζ^i $(i \in 1,2,3)$. Then, ζ^i = const. denotes a straight line parallel to the side of the triangle opposite the vertex i. The three associated lines through the point in question are now transformed via the mapping into curves in space lying on the middle surface of the shell element and passing through the corresponding point in space. Also the straight triangular mesh (based on ζ^i = const.) in the plane evolves into a curvilinear triangular mesh on the middle surface. Furthermore, the extension of the theory to large displacements gains in transparency if we imagine the finite element to be built up by an assembly of infinitesimal triangular subelements tangential to the middle surface.

It goes without saying that the development of the theory for the SHEBA element requires a good measure of differential geometry based on the above natural concepts. In the original theory limited space permitted only a partial account of these aspects and some of the details given were neither consistent nor concise. It is the purpose of this paper to remedy this situation with a complete discourse on the so-called natural differential geometry. It is evident that for all its intrinsic interest this theory is primarily a basic tool for a computer oriented analysis. The reader will observe the extremely compact and symmetrical expressions for the natural membrane and bending strains. In contrast to Love's classical theory no strains arise here under any rigid body movements. We note that the classical cartesian presentation is only required when setting up the final results and when considering the boundary and symmetry conditions.

It is of interest that the foundations of consistent shell theory developed in this paper for computer purposes achieve the same aim as the classical work of Koiter on a consistent analytical theory [4,5] .

2. Natural geometry in a plane

2.1 Notations

We summarise in this chapter the triangular or natural geometry in a plane which will be applied in the next chapter to the analysis of the so-called local triangles on a surface. First of all we choose a basic triangle and introduce the following notation

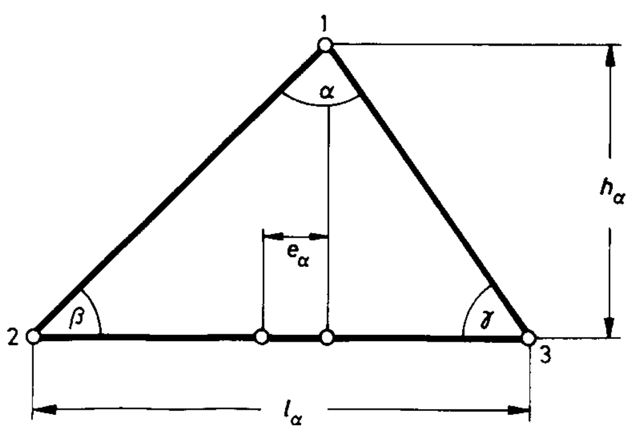

Fig. 2.1