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ANALYSE ET CONTRÔLE DE SYSTÈMES

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INSTITUT DE RECHERCHE D'INFORMATIQUE ET D'AUTOMATIQUE
DOMAINE DE VOLUCEAU - ROCQUENCOURT - B.P.105 - 78150 LE CHESNAY - TÉL.: 954 90 20

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TEXTES DES EXPOSÉS DU SÉMINAIRE ORGANISÉ PAR
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TABLE DES MATIÈRES

FLOOD STAGE FORECASTING IN RIVERS USING STOCHASTIC MODELS

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1. INTRODUCTION:

Forecasting by definition is the probable behaviour of the phenomenon, its accuracy depends upon the nature of the phenomenon, the nature of available data and the adequacy of the fitted model. Hydrologists are commonly required to forecast hydrologic phenomenon such as runoff, floods and rainfall in time. These phenomenon are complex in nature and its distribution is spatial and temporal.

In India, where the terrains are highly irregular the rainfall distribution is highly skewed during the year. Seventy-five per cent of rainfall is received during the monsoon months of June to September. The rainfall of the country as a whole is less variable from year to year (10 per cent of the annual mean rainfall) but it is extremely variable from month to month. This seasonal variability in Indian rainfall is responsible for causing floods or droughts in different parts of the country.almost every year.

Stochastic models have been widely used for generating synthetic data needed for working at optimal design and operation of water resources system (Lawrence and Kottegoda, 1977, Spolia and Chander, 1976). However, use of these models in hydrological forecasting has been limited.

Stochastic models for forecasting annual rainfall may give accurate prediction, but any suitable model required for quantitatively forecasting highly variable rainfall during monsoon months for short period of time when the flood havoc is maximum, shall require a large number of meteorological data collected at very close interval of time from densely distributed stations. Such studies could not be carried out in the absence of such data.

Chander and Spolia (1976), Chander (1975) have used ARMA models with stages recorded at upstream stations on the rivers or its tributaries as the inputs in forecasting flood stages at downstream stations on rivers. These forecasts are of great importance as they allow some time for evacuation of human as well as animal life and immovable property. Stage data has been used in these studies as these are the only data which can be easily and economically collected during floods.

Two case studies are described in this paper for flood stage prediction. In the first case, flood stages on the river Brahmaputra are predicted using upstream stage data from these tributaries as a multiple input AR model. The parameter of this model are estimated using Extended Kalman filter algorithm. In the second case, flood stage forecasts on the Wainganga river are worked out using rainfall as an input to the system. The parameters of the model in this case are estimated by recursive least square algorithm.

2. Case Study - I

In this case study, forecasting of stages on river Brahmaputra at Dibrugarh has been attempted using the observed gauge data up-stream, on the three major tributaries namely Dihang, Debang and Lohit. The hourly gauges during floods, are measured at Passighat on Dihang, Jiagaon and Teju on Lohit. The river bed in the tribu-taries and the main river suffers aggradation or degradation de-pending on the flows. A model based on gauges will need to be changed from flood to flood. Chander and Spolia (1976) got over this difficulty in their mutiple input autoregressive model by making use of the difference in gauges at the three upstream tribu-tary station as inputs to the autoregressive model of differences in in gauges at Dibrugarh.

The discrete linear, time invariant model proposed by them is given by equation 1.

$$g_{i+T,i} = A_1 g_{i,i-T} + \sum_{j=1}^{m} A_{2,j}\, h^{(J)}_{i-T_j+T,\, i-T_j} + \sum_{j=1}^{m} A_{3,j}\, h^{(j)}_{i-T_j,\, i-T_j-T} \qquad (1)$$

m = number of multiple inputs (tributaries in this case)
T = forecasting time
T_j = lag time between the upstream station on the jth tributary, and the forecasting station ($T \leq T_j$)

$h_{i-T_j+T,i-T_j}$ = difference in gauges at the upstream station on the j^{th} tributary between $(i-T_j+T)^{th}$ and $(i-T_j)^{th}$ instant.

$g_{i,i-T}$ = difference in gauges at i^{th} and $(i-T)^{th}$ instant of time at the forecasting station.

A_1, A_{2j}, A_{3j} are the parameters.

Equation 1 can also be written as

$$y(k) = A_1 y(k-1) + \sum_{j=1}^{m} A_{2,j}\, w^{(J)}(k-T_j) + \sum_{j=1}^{m} A_{3,j}\, w^{(J)}(k-1-T_j) \qquad (2)$$

where $y(k)$ and $y(k-1)$ are the outputs at time k and k-1 respectively and $w(k-T_j)$ and $w(k-1-T_j)$ are the inputs from tributaries at times $(k-T_j)$ and $(k-1-T_j)$ respectively.

2.1 Estimation of parameters

The parameters A_1, $A_{2,j}$ and $A_{3,j}$ can be estimated by least square method using T_j's for each of the m tributaries and observed input and output data. The observed input and output data were plotted and an estimate of delay time T_j for each tributary was made on the basis of time difference between the peak of the jth tributary and the peak at the output station. Delay time T_j of 12. 12 and 14 hours was estimated for Passighat, Tezu and Jiagaon respectively on this basis. Knowing T_j's, the optimal estimates of the parameters are given by

$$\hat{\theta}(k) = |H'(k)\, H(k)|^{-1} H'(k)\, \underline{Y}(k) \qquad (3)$$

where $\underline{\theta}(k)$ is a $(2m+1)\times 1$ vector containing parameters

$$= \{A_1, A_{21} - - - - A_{2m}\, A_{31} - - - - A_{3m}\}'$$

Let N be the number of observations

$H(k) = N \times (2m+1)$ matrix of observed data

and $\underline{Y}(k)$ = $(N \times 1)$ vector, containing outflows at Dibrugarh,

$$Y(k) = |y(k),\ h(k-1) ---- y(k-N)|'$$

If $P(k) = |H'(k)H(k)|^{-1}$

then

$$\hat{\underline{\theta}}(k) = P(k)\, H'(k)\underline{Y}(k) \tag{4}$$

The parameters estimates generated by equation (3) for the model given by equation (1) need be **updated** as new observations become available.

System updating can be done using recursive least spares (Mendel 1973) or extended Kalman filter (Jazwinkky 1970).

2.2 Parameter Estimation using recursive least square:

Let the updated estimates of the parameters be $\hat{\theta}(k+1)$

Then $\hat{\underline{\theta}}(k+1) = P(k+1)\, H(k+1)\, \underline{Y}(k+1)$ (5)

Now

$$\underline{Y}(k+1) = \left|\frac{y(k+1)}{\underline{Y}(k)}\right| \quad \text{and } H(k+1) = \left|\frac{h(k+1)}{H(k)}\right|$$

Equation (5) can be rearranged as

$$\hat{\underline{\theta}}(k+1) = P(k+1)\underline{h}'(k+1)\left|y(k+1) - \underline{h}(k+1)\,\hat{\underline{\theta}}(k)\right| + \hat{\underline{\theta}}(k) \tag{6}$$

Equation (6) can be used for updating parameters, knowing $\hat{\theta}(k)$, $\underline{h}(k+1)$, and $y(k+1)$ but needs two inversions to obtain $P(k+1)$. The computation can be further simplified using matrix Lemma (Mendel 1973).

$$\hat{\underline{\theta}}(k+1) = \hat{\underline{\theta}}(k) + P(k)\underline{h}'(k+1)\,|h(k+1)P(k)h'(k+1)+1|^{-1}\,(y(k+1) - \underline{h}(k+1)\,\hat{\underline{\theta}}(k)) \tag{7}$$

Equation (7) reduces the computation as $(hPh'+1)$ is scalar quantity and can be easily used for upgrading the parameters.

2.3 Parameter Estimation using Extended Kalman Filter

To use extended Kalman filtering equation (2) is written in the standard form as an input-output noise system as

$$\underline{X}(k+1) = \alpha(k)\,\underline{X}(k) + \beta(k)\underline{U}(k) + \underline{\nu}(k)$$

$$\underline{Y}(k+1) = H(k+1)\,X(k+1) + \underline{V}(k+1) \tag{8}$$

where $U(k)$ is an accessible input vector of data form tributaries and $\nu(k)$ is the n-vector noise sequence which takes into account any modelling error or the random inputs to the system.- $V(k)$ is the measurement error vector and $Y(k)$ is the noisy observation of $X(k)$.

Assuming the parameters are randomly varying which can be written in the form

$$\underline{\theta}(k) = \underline{\theta}(k-1) + \underline{\xi}(k-1) \tag{9}$$

where $\underline{\theta}(k)$ forms a vector of unknown elements of matrices $\beta(k)$ and $\alpha(k)$. ξ is a random sequence having zero mean white gaussian with known covariance S. The parameters $\underline{\theta}(k)$ were estimated using the standard extended Klaman filter algorithm (Jazwinsky 1970). The covariance estimates of noisy sequences **are estimated** using adaptive estimation algorithm given by Sage and Husa (1969).

The updated parameters with every new observation in equation 2 were used for three floods shown in figure 1-1, 1-2 and 1-3. These forecasts were made at 4 hours and 8 hours intervals.

The results were compared with the observed hydrographs. It is concluded that 4 hour forecasts are nearer to the observed hydrograph than the 8 hour forecasts.

2.5 Case Study - 2.

In this study stage forecasting of river Vainganga is attempted using rainfall data. Rainfall is measured at seven different stations, namely i) Seoin Chapra, ii) Kunda, iii) Mohgaon, iv) Karabdol, v) Fijna, vi) Gopalganj, and vii) Seoni, in this catchment. The weightee average rainfall was calculated using the Theissen's method and was used as input to the model.

$$y(t) = \phi_1 y(t-T) + \phi_2 y(t-2T) + \theta_0 + \theta_1 w(t-T) + \theta_2 w(t-2T) + \theta_3 w(t-3T) \quad (10)$$

where

$y(t)$ = River stage at time t
$w(t-T)$ = Weighted average rainfall at time $(t-T)$

T was assumed to be 3 hours and 3 hours ahead of forecasts were made in this case.

The parameters were initially estimated using first 25 observations of a multipeaked flood observed on 29-7-68 to 6-8-68 (Figure 2-1). The model was then used to forecast the stages for the remaining period of the flood.

The observed and the predicted stage hydrograph using the sa same parameters are also shown in figure 2-1.

In order to find whether the basis was well represented by a time variant or time invariant model with the structure given in equation 10, two floods observed during the years 1972 and 1973 were analysed and 3 hours ahead forecasts using constant parameters and time invariant model with updated parameters (equation 7)) were computed. The computed and observed hydrographs were shown in figure 2-2 and 2-3 for both cases. The mean square criterion was used for evaluating the models and it was found that recursive meast square algorithm reduced the mean square error value considerably, for 1972 it was reduced from 0.1663 to 0.0866 and for 1973 flood, from 0.245 to 0.01395, thus implying that the forecasts are better using updated parameter in a model.

Conclusions

It has been shown in this paper that recursive least square and extended Kalman filters can be used for estimation of parameters of the model used in flood stage forecasting. This method becomes particularlyuseful as they do not require data storage and thus help implementation of such models on small mini process control computers.

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