Séminaires IRIA

ANALYSE ET CONTRÔLE DE SYSTÈMES

1979

PUBLIÉ PAR

IRIA

INSTITUT NATIONAL DE RECHERCHE
EN INFORMATIQUE ET EN AUTOMATIQUE
DOMAINE DE VOLUCEAU - ROCQUENCOURT - B.P. 105 - 78150 LE CHESNAY - TÉL.: 954 90 20

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TEXTES DES EXPOSÉS DU SÉMINAIRE ORGANISÉ PAR
L'INSTITUT DE RECHERCHE D'INFORMATIQUE ET D'AUTOMATIQUE (IRIA)

ROCQUENCOURT

Février 1979 - Octobre 1979

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TABLE DES MATIÈRES

SOME CLASSES OF TWO-PARAMETER MARTINGALES

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Summary

The most natural definition of a two-parameter martingale is, perhaps, the process satisfying $E(X_{(s_2,t_2)}|\mathcal{F}_{(s_1,t_1)}) = X_{(s_1,t_1)}$ whenever $s_2 \geq s_1$ and $t_2 \geq t_1$. As is well known not all the properties of one-parameter martingale are inherited by two-parameter martingales under this definition and this leads to the introduction of other classes of martingales with the same partial ordering which are either weaker (e.g., weak martingales, 1- and 2-martingales) or stronger (e.g., strong martingales, martingales of path independent variation) than the natural class of martingales. Strong martingales were introduced in [1] and shown there to play an important role in the theory of two-parameter martingales and stochastic integration, martingales of path independent variation were defined in [2] as follows: A continuous square integrable martingale **M** is said to be of path independent variation if the quadratic variation of **M**, as a one-parameter martingale along every increasing path depends on the initial and end points of the path only. It was shown in [1] that martingales of path independent variation on the sigma fields generated by the two-parameter Wiener processes are strong martingales and it was shown that in a certain special case this is actually so. The work presented here was motivated by the relations between strong and path independent martingales, it considers the problem of characterizing strong martingales by sample function properties and gives a partial answer to this problem. A class of martingales which will be called "martingales of direction independent variation" ("martingales of orthogonal increments" may be more appropriate) is introduced as follows: Let **M** be a continuous square integrable martingale and let $Y_z = \int_{R_z} I_a(\xi) dM_\xi$ where $z = (z_1, z_2)$, $a = (a_1, a_2)$, $\xi = (\xi_1, \xi_2)$, $I_a(\xi) = 0$ if either $\xi_1 < a_1$ or $\xi_2 < a_2$ and $I_a(\xi) = 1$ otherwise. **M** is said to be of direction independent variation if Y_z is of path independent variation for all **a** in $\mathbb{R}^2_+$. The class of direction independent martingales includes the class of strong martingales and is included in the class of martingales of path independent variation. Like the class of path independent martingales, the class of direction independent

martingales is also characterized by a sample function property. On the other hand, martingales of direction independent variation share with strong martingales several important properties so that results which were obtained for strong martingales hold for direction independent martingales. In particular, the requirement that **M** be a strong martingale in the definition of the stochastic integral of the second type, $\int\int \psi dMdM$, ([1]) can be replaced by the requirement that **M** is of direction independent variation. Finally, direction independent martingales on the sigma fields generated by the Wiener process are strong martingales.

References

[1] Cairoli, R. and Walsh, J. B., 1975. Stochastic integrals in the plane. Acta Math., 134, pp. 111-183.

[2] Wong, E. and Zakai, M., 1974. Martingales and stochastic integrals for processes with a multi-dimensional parameter. Z. für Wahr. verw. Geb., 29, pp. 109-122.

BOUNDARY METHODS GENERAL THEORY

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INTRODUCTION

Boundary methods for treating numerically partial differential equations associated with many problems of Science and Engineering are receiving attention at present. There are several procedures for formulating such problems. Most frequently these methods have been based on Maxwell Betti's integral equation [1]-[5], but alternative integral representations have been considered by some authors [6]-[9]. Another approach is to use a suitable complete set of solutions to approximate any other one. This formulation is frequently called Kupradze's functional equations. Its theoretical foundation can be traced back to the method of Fischer-Riesz equations [10], and Kupradze [11] has given a procedure for constructing the necessary complete system when a fundamental solution of the differential equation considered, is known.

In many applications, a part of the region is treated numerically by means of finite elements and the sought solutions are required to be such that can be continued smoothly into neighboring regions as solutions of given differential equations. Such kind of boundary conditions will be called continuation type restrictions [12]. It is possible and useful to formulate variational principles which account for them and which only involve the region treated numerically. Again, variational principles of this type can be applied when a complete set of solutions in the neighboring regions is available (for an example, see [13]).

The theory of connectivity, is an abstract theory of problems subjected to linear restrictions or constraints recently developed by the author [12], [14]-[19], which supplies a convenient general framework for the formulation of such variational principles and the discussion of questions of completeness. The purpose of this lecture is to give a brief description of that theory and examples of its applicability. Up to now it is only applicable to formally symmetric operators, but is being revised to extend it to arbitrary linear operators. The detailed proofs of most of the results to be presented are contained in [12] and [20].

THE GENERAL FRAMEWORK

In this paper linear operators such as $P:D\to D^*$ will be considered, where D is a linear space with coefficients in the field $\mathcal{F}$ of real, or alternatively, complex numbers, and D^* its algebraic dual. For every $u\in D$, the value of $P(u)\in D^*$ at $v\in D$ will be denoted by $\langle P(u),v\rangle\in\mathcal{F}$; the latter defines a bilinear functional and the inner parenthesis will be deleted. The adjoint operator $P^*:D\to D^*$ always exists and satisfies $\langle P^*u,v\rangle=\langle Pv,u\rangle$.

There are many problems that can be cast in the following framework.

Definition 2.1. Consider $P:D\to D^*$ *and a subspace* $I\subset D$. *Given* $U\in D$ *and* $V\in D$, *and element* $u\in D$ *is said to be a solution of the problem with linear restrictions or constraints, when*

$$Pu = PU \quad \text{and} \quad u - V\in I. \qquad (2.1)$$

As an example, consider the operator $P:D\to D^*$ defined by

$$\langle Pu,v\rangle = \int_R v\,\nabla^2 u\,d\underset{\sim}{x} \qquad (2.2)$$

where region R is illustrated in Figure 1. There are many ways in which D can be taken, because it is only required to be a linear space without any further structure. For definiteness, one may think of D as being the Sobolev space $H^s(R)$; $s\geq 2$ [21]. Define the linear subspace $I\subset D$ by

$$I = \{u \in D \mid u = 0, \quad \text{on } \partial R\}. \qquad (2.3)$$

Then, problem (2.1) is Poisson's equation

$$\nabla^2 u = \nabla^2 U = f_R \;; \quad \text{on} \quad R \qquad (2.4a)$$

subjected to boundary conditions of Dirichlet type

$$u = V = f_{\partial R} \;; \quad \text{on} \quad \partial R\,. \qquad (2.4b)$$

Define

$$A = P-P^* \;;\; N = N_A = \{u\in D \mid Au=0\} \qquad (2.5)$$

In applications the operator N introduces a classification of boundary conditions; for example, when P is given by (2.2), $N=\{u\in D \mid u = \partial u/\partial n,\ \text{on } \partial R\}$.

Definition 2.2. A subspace $I\subset D$ *on which* P *is commutative and such that* $N\subset I$ *is said to be regular for* P. *It is completely regular, if in addition*

$$\langle Au,v\rangle = 0 \quad \forall\ v\in I \;\Rightarrow\; u\in I \qquad (2.6)$$