Computational Statistics (2022) 37:591-611 https://doi.org/10.1007/s00180-021-01142-y

## ORIGINAL PAPER

## Stochastic functional linear models and Malliavin calculus Ruzong Fan1 · Hong-Bin Fang1

Received: 30 September 2020 / Accepted: 4 August 2021 / Published online: 27 August 2021 © The Author(s), under exclusive licence to Springer-Verlag GmbH Germany, part of Springer Nature 2021

## Abstract

In this article, we study stochastic functional linear models (SFLM) driven by an underlying square integrable stochastic process X(t) which is generated by a standard Brownian motion. Utilizing the magnificent Itô integrals and Malliavin calculus, X(t)is expanded into a summation of orthogonal multiple integrals, i.e., Wiener-Itô chaos expansions, which is the counterpart of the Taylor expansion of deterministic functions. Based on the expansion, we show that the fourth moments of linear functionals of underlying stochastic process X(t) are bounded by the square of their second moments when X(t) is a finite linear combination of multiple Itô integrals. Therefore, an optimal minimax convergence rate in mean prediction risk of SFLM is valid if eigenvalues of related linear operators are of order k-2r by using results in literature when the underlying process X(t) is a linear combination of multiple Itô integrals. A sufficient and necessary condition of finite fourth moment of random functions of multiple Itô integrals is proved, which is a key condition in methodology and convergence rates of functional linear regressions. Our results show that the optimal minimax convergence rate in mean prediction risk can be applied to the class of linear combination of multiple Itô integrals which are not necessarily Gaussian processes. Moreover, the sufficient and necessary condition of finite fourth moment for multiple Itô integrals can be directly applied to showmethodology and convergence rates of functional linear models. Using the theory of stochastic analysis, one may construct a reproducing kernel Hilbert space (RKHS) associated with a square integrable stochastic process to facilitate analysis of functional data.

**Keywords** Itô integrals · Malliavin calculus · Wiener-Itô chaos expansions · Functional linear models · Minimax · Optimal convergence rate · Reproducing kernel Hilbert space · Smoothing splines · Sobolev space