A Calculational Deductive System for Linear Temporal Logic

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This article surveys the linear temporal logic (LTL) literature and presents all the LTL theorems from the survey, plus many new ones, in a calculational deductive system. Calculational deductive systems, developed by Dijkstra and Scholten and extended by Gries and Schneider, are based on only four inference rules—Substitution, Leibniz, Equanimity, and Transitivity. Inference rules in the older Hilbert-style systems, notably modus ponens, appear as theorems in this calculational deductive system. This article extends the calculational deductive system of Gries and Schneider to LTL, using only the same four inference rules. Although space limitations preclude giving a proof of every theorem in this article, every theorem has been proved with calculational logic.

CCS Concepts: • **Theory of computation → Modal and temporal logics**;

Additional Key Words and Phrases: Calculational logic, equational logic, linear temporal logic

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1 INTRODUCTION

Linear temporal logic (LTL) has application to proof of correctness for concurrent programs. Many concurrent programs, such as operating systems and embedded systems that control physical equipment, are nonterminating by design. Consequently, proof techniques that depend on proving the correctness of postconditions on program termination do not apply. LTL, however, can be used to prove desirable program traits such as freedom from deadlock.

Most treatments of LTL consist of cursory introductions in one or two chapters of graduate-level textbooks [2, 20, 21, 24]. While many LTL theorems are common in the different treatments, each treatment has theorems that are unique to it. This survey is a comprehensive collection of all the LTL theorems that we have found in the literature, together with many new theorems, all of which are presented in an axiomatic logic system. It serves as an introduction to LTL and should be accessible with a prerequisite only of the standard propositional and predicate logic at the undergraduate level.

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