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4-edge-coloring graphs of maximum degree 3 in linear time

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Abstract

We present a linear time algorithm to properly color the edges of any graph of maximum degree 3 using 4 colors. Our algorithm uses a greedy approach and utilizes a new structure theorem for such graphs. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Graph coloring is one of the most fertile and well-studied subjects in Graph Theory. An evidence of this fact may be found by browsing through the list of solved and unsolved problems in a comprehensive book [8] on graph coloring problems. The most general problem in the field is vertex coloring, since many coloring problems can be reduced to it. In the vertex coloring problem, we want to use the least number of colors to color the vertices of a graph, one color per vertex, in such a way that no two adjacent vertices are assigned the same color. This least number of colors is called the chromatic number χ of the graph. We are interested in a related edge coloring problem. In this problem, we want to use the least number of colors to color the edges of a graph, one color per edge, in such a way that no two adjacent edges are assigned the same color. This least number of colors is called the chromatic index χ' of the graph. A theorem of Vizing states that the chromatic index of a simple graph is at most one larger than its maximum degree Δ [15,11]. Therefore the chromatic index of any simple graph is either Δ or $\Delta + 1$. Determining the true value proves to be an NP-complete problem, as shown by Holyer [7]. In fact, Holyer [7] shows that the problem remains NP-complete even when restricted to *cubic* graphs, those graphs whose every vertex is incident with exactly three edges. As observed in the survey paper [6] on cubic graphs, *cubic graphs often seem to be the simplest class of graphs for which a problem remains as difficult to solve as on a general graph.* By studying the problem when restricted to cubic graphs we may gain insight into why the problem is difficult. Specific subclasses of cubic graphs have been investigated and have acquired their own special significance. A good example is the **SNARK**s, those bridgeless cubic graphs with chromatic index four. A survey on **SNARK**s is given in [16]. The class of graphs of interest to us properly contains the class of cubic graphs.

We will be concerned with graphs of maximum degree 3 from now on. By Holyer [7], the problem of

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