BER Analysis of MPSK Space-Time Block Codes With Differential Detection

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***Abstract*—Closed-form expressions of the bit error rate (BER) are derived for space-time block codes based on Alamouti's scheme and utilizing M-ary phase shift keying modulation with noncoherent differential encoding/decoding. The analysis is carried out for the flat, block-fading Rayleigh channel, and the BER expression is an approximation for high signal to noise ratio. Theoretical results are validated by simulations for BPSK and QPSK modulations.**

***Index Terms*—Bit error rate (BER), differential detection, space-time block code (STBC).**

I. Introduction

THIS letter is motivated by the observation that for the special case of space-time block codes (STBCs) based on Alamouti's scheme [1], it is possible to obtain a closed-form expression for the bit error rate (BER). A closed-form BER expression would serve as an attractive alternative to previously derived bounds for evaluating performance of STBC [2], [3]. It is widely understood that Alamouti's scheme has a performance loss of 3 dB with respect to two-branch maximum ratio combining (MRC). It follows that the BER for Alamouti's scheme with coherent detection can be obtained directly from the result on receive diversity [4]. Here, we are concerned with the BER analysis for STBC with (noncoherent) differential detection. While the procedure for deriving the BER applies to any M-ary phase shift keying (MPSK) signaling, binary PSK (BPSK) and quadrature PSK (QPSK) examples are worked out in detail.

II. System Model

Assume a communication system with two transmit antennas and Q receive antennas operating over a flat, Rayleigh channel. The transmission scheme generates blocks of two symbols transmitted by each of the two antennas. With each block, we associate a time index $k = 1, \ldots, K$. The received signal at time index k, time slots 1, 2 within the block, and receive antenna q, $1 \leq q \leq Q$, is given by

$$\begin{bmatrix} r_{1,k}^{(q)} \\ r_{2,k}^{(q)} \end{bmatrix} = \begin{bmatrix} c_k^{(1)} & c_k^{(2)} \\ -c_k^{(2)*} & c_k^{(1)*} \end{bmatrix} \begin{bmatrix} h_1^{(q)} \\ h_2^{(q)} \end{bmatrix} + \begin{bmatrix} n_{1,k}^{(q)} \\ n_{2,k}^{(q)} \end{bmatrix} \quad (1)$$

where $h_1^{(q)}$, $h_2^{(q)}$ are the path gains from transmit antenna 1 and 2, respectively to receive antenna q. Path gains are modeled as complex Gaussian random variables with zero-mean and variance 1/2 per dimension. The channel is assumed block-fading, i.e., fixed over the duration of K blocks. Noise samples $n_{1,k}^{(q)}$, $n_{2,k}^{(q)}$ are zero-mean complex Gaussian random variables with variance $N_0/2$ per dimension. The symbols $c_k^{(i)}$ are differentially encoded, transmitted by antennas $i = 1, 2$. Without loss of generality, we assume that the symbols amplitude is $1/\sqrt{2}$ (such that the signal to noise ratio per symbol is $1/2N_0$). The superscript '$*$' denotes complex conjugation.

The differential STBC scheme analyzed in this letter is the one recently proposed by Tarokh and Jafarkhani [5] based on Alamouti's transmit diversity scheme. The message matrix is represented by the unitary 2×2 matrix

$$\mathbf{S}_k = \begin{bmatrix} s_k^{(1)} & s_k^{(2)} \\ -s_k^{(2)*} & s_k^{(1)*} \end{bmatrix} \quad (2)$$

where the symbols $s_k^{(i)}$ belong to an MPSK constellation. The message matrix $\mathbf{S}_k$ is differentially encoded by a procedure resembling standard single-antenna DPSK. To initialize transmission, the transmitter sends a code unitary matrix $\mathbf{C}_0$, for example

$$\mathbf{C}_0 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}. \quad (3)$$

The differentially encoded message $\mathbf{C}_k$ at time k, $k \geq 1$, is obtained by multiplying the codeword at time $k-1$, $\mathbf{C}_{k-1}$ by the current message $\mathbf{S}_k$, namely $\mathbf{C}_k = \mathbf{S}_k \mathbf{C}_{k-1}$. Note that the codeword $\mathbf{C}_k$ has the same unitary property as the message matrix $\mathbf{S}_k$. In the absence of noise, the messages $\mathbf{S}_k$ can be decoded from $\mathbf{C}_k \mathbf{C}_{k-1}^{\dagger} = \mathbf{S}_k \mathbf{C}_{k-1} \mathbf{C}_{k-1}^{\dagger} = \mathbf{S}_k$, where '$\dagger$' denotes the Hermitian operation.

At the receiver, at the output of each antenna, we form the matrices $\mathbf{R}_k^{(q)}$, $1 \leq q \leq Q$,

$$\mathbf{R}_k^{(q)} = \mathbf{C}_k \mathbf{H}^{(q)} + \mathbf{N}_k^{(q)} \quad (4)$$

where

$$\mathbf{R}_k^{(q)} = \begin{bmatrix} r_{1,k}^{(q)} & -r_{2,k}^{*(q)} \\ r_{2,k}^{(q)} & r_{1,k}^{*(q)} \end{bmatrix}, \quad \mathbf{C}_k = \begin{bmatrix} c_k^{(1)} & c_k^{(2)} \\ -c_k^{(2)*} & c_k^{(1)*} \end{bmatrix},$$

$$\mathbf{H}^{(q)} = \begin{bmatrix} h_1^{(q)} & -h_2^{(q)*} \\ h_2^{(q)} & h_1^{(q)*} \end{bmatrix}, \quad \mathbf{N}_k^{(q)} = \begin{bmatrix} n_{1,k}^{(q)} & -n_{2,k}^{(q)*} \\ n_{2,k}^{(q)} & n_{1,k}^{(q)*} \end{bmatrix}.$$

Note that this construction ensures that $\mathbf{R}_k^{(q)} \mathbf{R}_k^{(q)\dagger}$ is a diagonal matrix with equal entries. For Q receive antennas, the signal model is

$$\mathbf{R}_k = \mathbf{D}_k \mathbf{H} + \mathbf{N}_k, \quad (5)$$

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