A "Better Than" Nyquist Pulse

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***Abstract*—A novel ISI-free pulse is presented that has smaller maximum distortion, a more open receiver eye, and a smaller probability of error in the presence of symbol timing error than the Nyquist pulse for the same excess bandwidth.**

I. Introduction

THE NYQUIST pulse with excess bandwidth, α, is specified by its frequency spectrum

$$S(f)=\begin{cases}1, & 0\le f\le B(1-\alpha)\\ \frac{1}{2}\left\{1+\cos\left(\frac{\pi}{2B\alpha}(f-B(1-\alpha))\right)\right\}, & B(1-\alpha)\le f\le B(1+\alpha)\\ 0, & B(1+\alpha)\le f\end{cases} \tag{1}$$

where B is the bandwidth corresponding to symbol repetition rate $T=1/(2B)$ and its corresponding (scaled) time function [1]

$$p_{NY}(t)=\mathrm{sinc}(t/T)\frac{\cos(2\pi\alpha t/T)}{1-4\alpha^2t^2/T^2}. \tag{2}$$

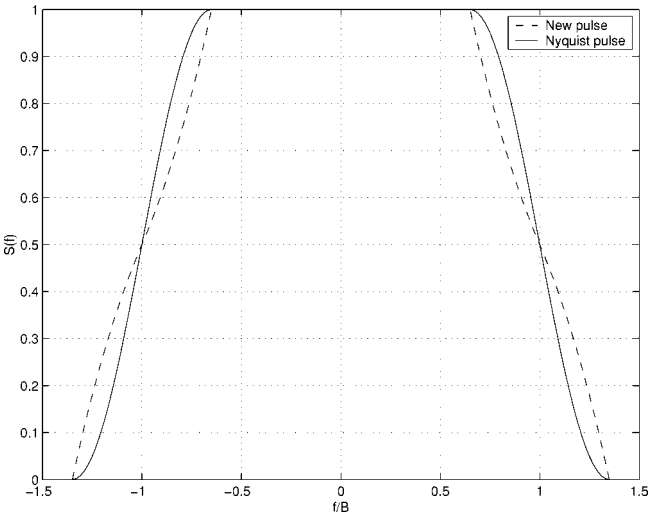
Examination of (2) indicates that the tails of the Nyquist pulse for $\alpha>0$ decay asymptotically as t^{-3}, as is well known. The Nyquist pulse is widely used in modem design and is the benchmark pulse in communication theory [2]. A novel pulse is specified by the frequency spectrum

$$S(f)=\begin{cases}1, & 0\le f\le B(1-\alpha)\\ \exp\left\{\frac{\ln 2}{\alpha B}\left[f-B(1-\alpha)\right]\right\}, & B(1-\alpha)\le f\le B\\ 1-\exp\left\{\frac{\ln 2}{\alpha B}\left[B(1+\alpha)-f\right]\right\}, & B\le f\le B(1+\alpha)\\ 0, & B(1+\alpha)\le f\end{cases} \tag{3}$$

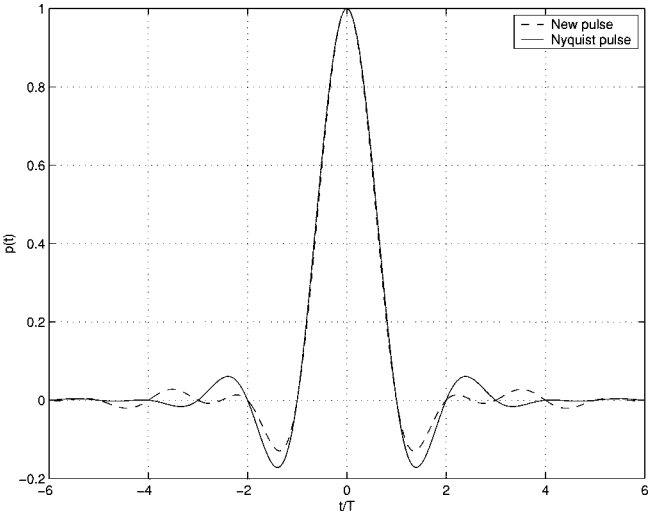
with corresponding (scaled) time function

$$p(t)=2B\mathrm{sinc}(2Bt)\cdot\frac{4\beta\pi t\sin(2\pi B\alpha t)+2\beta^2\cos(2\pi B\alpha t)-\beta^2}{4\pi^2t^2+\beta^2} \tag{4}$$

where $\beta\triangleq(\ln 2/\alpha B)$. Fig. 1 shows the frequency spectrum and time functions, respectively, of the new pulse for $\alpha=0.35$. It can be proved using (4) that the tails of the new pulse for $\alpha>0$ decay asymptotically as t^{-2}. Note that an infinite sequence of pulse tails is absolutely summable for both pulses, though the asymptotic rate of decay is greater for the Nyquist pulse. Despite the latter fact, the new pulse is "better than" the Nyquist pulse as shown in the next section.



(a)



(b)

Fig. 1. Frequency and time characteristics of the new and Nyquist pulses for an excess bandwidth $\alpha=0.35$. (a) Frequency domain. (b) Time domain.

II. Transmission Properties

Fig. 2 shows receiver eye diagrams for the new and Nyquist pulses. Observe that the eye for the new pulse is more open. This is highlighted by superimposing the inner boundary of the

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