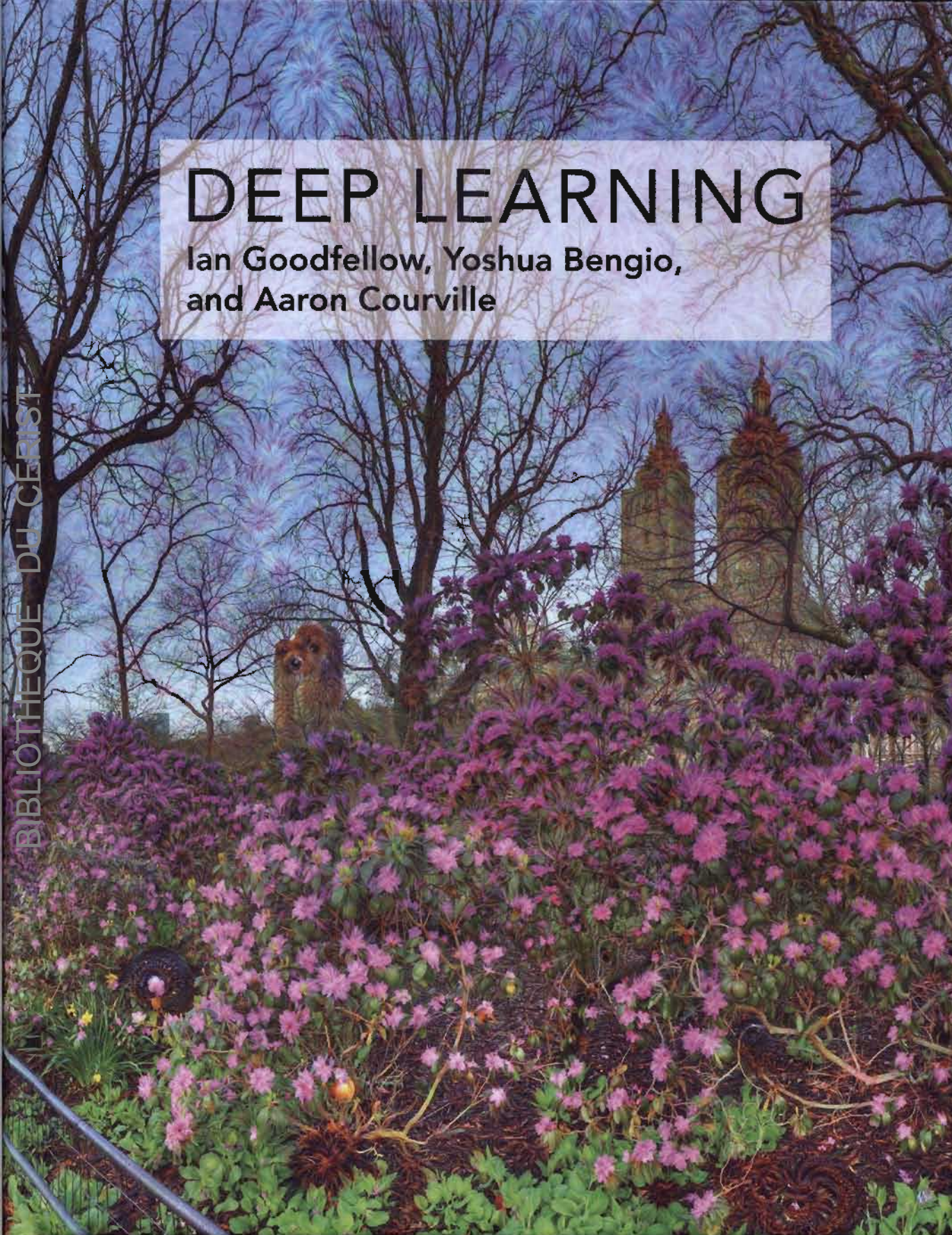
DEEP LEARNING
Ian Goodfellow, Yoshua Bengio,
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Deep Learning

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